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带有时空时滞的混合扩散竞争系统的行波解

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摘 要:本文主要研究一类带有时空时滞的混合扩散竞争系统行波解的存在性。首先,为了消除时空时滞的影响,根据时空时滞中核函数的特性,把带有时空时滞的由两个方程耦合而成的扩散系统转化为无时滞的由四个方程耦合而成的扩散系统。其次,通过变换将系统转化为单调系统。最后,利用上下解方法和不动点定理证明行波解的存在性。

关键词:反应扩散系统;上下解;非局部时滞
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Traveling Wave Solutions for Mixed Diffusive Competition Systems with Spatio-temporal Delays

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Abstract: In this paper, the existence of traveling wave solutions of mixed diffusive competition systems with spatio-temporal delays was studied. Firstly, in order to eliminate the influence of spatio-temporal delays, the delayed diffusion system with two equations was changed into the non-delay diffusion system with four equations based on the property of the kernels in spatio-temporal delays. Then the system was further changed into the monotone system. Finally, the main results were obtained by using the super- and sub-solutions method and the fixed point theorem.

Key words: reaction diffusion system; super- and sub-solutions; nonlocal delays

0 引言

在实际问题中,如果考虑两个不同种群在同一个环境下生活,随着时间的演化,由于环境承载力有限,这两个种群之间会产生相互竞争,这种竞争关系可用经典的Lotka-Volterra反应扩散竞争系统刻画,但经典的反应扩散系统无法反映种群在某一位置某一时刻的密度受之前时刻密度的影响,为此,在原有系统中引入了离散时滞,有限分布时滞等。Britton[1-2]在此基础上进一步提出了时空时滞,即一类非局部时滞,这类时滞可以更好地解决建模中扩散和时滞的相互独立性,并具有重要的生物意义,同时,研究离散时滞系统行波解存在性的方法无法直接应用于研究带有时空时滞系统行波解的存在性,因此,时空时滞的引入也带来数学研究上的新困难。基于此,本文将克服

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时空时滞的影响并研究如下Lotka-Volterra混合扩散竞争系统行波解的存在性,

$$\begin{cases}(u_1)_t=(u_1)_{xx}+u_1(1-u_1-a_1K*u_2),\\(u_2)_t=d(J*u_2-u_2)+ru_2(1-a_2u_1-u_2),\end{cases}\tag{1}$$

其中$u_i(i=1,2)$为种群密度,a_1,a_2,d,r为正常数,满足假设

(A_1) $0<a_1<1<a_2$,

核函数J满足如下假设

(A_2) $J\in C(R),J(x)=J(-x)\geqslant 0,x\in R,\int_R J(x)\mathrm{d}x=1,\int_R|x|^iJ(x)\mathrm{e}^{\lambda x}\mathrm{d}x<\infty,i=1,2,3,\lambda\in R$,

核函数K为$K(x,t)=G(x,t)k(t)$, $G(x,t)=\frac{1}{\sqrt{4\pi t}}\mathrm{e}^{-\frac{x^2}{4t}}$, $k(t)=\frac{t}{\tau^2}\mathrm{e}^{-\frac{t}{\tau}},\tau>0$为时滞,卷积$K*u_2$为$(K*u_2)(x,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{t}K(x-y,t-s)u_2(y,s)\mathrm{d}s\mathrm{d}y$,不难验证,函数$G(x,t)$满足初值问题$\begin{cases}G_t=G_{xx},\\G(x,0)=\delta(x),\end{cases}$其中$\delta(x)$为Dirac函数,并且$\int_{-\infty}^{+\infty}\int_0^{\infty}K(x,t)\mathrm{d}t\mathrm{d}x=1$, $\lim_{\tau\to 0}(K*u_2)(x,t)=u_2(x,t)$。

当$J(x)=\delta(x)-\delta''(x)$时,其中,$\delta(x)$为Dirac函数,系统(1)变成

$$\begin{cases}(u_1)_t=(u_1)_{xx}+u_1(1-u_1-a_1K*u_2),\\(u_2)_t=d(u_2)_{xx}+ru_2(1-a_2u_1-u_2)。\end{cases}\tag{2}$$

进一步,当$\tau\to 0^+$时,系统(2)变成经典的Lotka-Volterra反应扩散竞争系统,

$$\begin{cases}(u_1)_t=(u_1)_{xx}+u_1(1-u_1-a_1u_2),\\(u_2)_t=d(u_2)_{xx}+ru_2(1-a_2u_1-u_2)。\end{cases}\tag{3}$$

当假设(A_1)成立时,系统(3)有三个非负平衡点(0,0),(1,0),(0,1),并且在对应的常微分系统中,(1,0)是稳定的,(0,1)是不稳定的。因此,自然考虑种群u_1入侵种群u_2,而这一入侵现象可以用系统(1)—(3)连结(0,1)和(1,0)的行波解表示。对于系统(3)连结(0,1)和(1,0)的行波解存在性研究已有大量结果,可参考文献[3-4]等。

当$K(x,t)=G(x,t)k(t),k(t)=\frac{1}{\tau}\mathrm{e}^{-\frac{t}{\tau}}$时,此时,核函数称为弱核[5]。当$d=1$时,Hou等[6]利用单调迭代的方法证明了系统(2)连结(0,1)和(1,0)的行波解的存在性。之后,当$d>0$时,Chang等[7]利用上下解方法和谱分析方法证明了行波解的存在性和稳定性。进一步,对于带有混合扩散的系统(1),Hao和Zhang[8]利用上下解方法和加权能量方法证明了行波解的存在性和稳定性。当$K(x,t)=G(x,t)k(t)$, $k(t)=\frac{t}{\tau^2}\mathrm{e}^{-\frac{t}{\tau}}$时,核函数称为强核[5],系统(2)连结(0,1)和(1,0)的行波解的相关研究可以参考文献[9]。对于时滞系统的行波解存在性的研究可以参考文献[10-12]。

受文献[5-9]的启发,在本文,当核函数为强核时,我们采用文献[7]的上下解构造方法证明系统(1)连接(0,1)和(1,0)的行波解的存在性。为此,令

$$v_1(x,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{t}\frac{1}{\tau\sqrt{4\pi(t-s)}}\mathrm{e}^{-\frac{(x-y)^2}{4(t-s)}}\mathrm{e}^{-\frac{t-s}{\tau}}u_2(y,s)\mathrm{d}s\mathrm{d}y,$$

$$v_2(x,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{t}\frac{\sqrt{t-s}}{\tau^2\sqrt{4\pi}}\mathrm{e}^{-\frac{(x-y)^2}{4(t-s)}}\mathrm{e}^{-\frac{t-s}{\tau}}u_2(y,s)\mathrm{d}s\mathrm{d}y。$$

不难验证v_1,v_2满足

$$(v_1)_t=(v_1)_{xx}+\frac{1}{\tau}(u_2-v_1),(v_2)_t=(v_2)_{xx}+\frac{1}{\tau}(v_1-v_2)。$$

为方便起见,令$\rho=\frac{1}{\tau}$。因此,系统(1)连结(0,1)和(1,0)的行波解等价于系统

$$\begin{cases}(u_1)_t=(u_1)_{xx}+u_1(1-u_1-a_1v_2),\\(u_2)_t=d(J*u_2-u_2)+ru_2(1-a_2u_1-u_2),\\(v_1)_t=(v_1)_{xx}+\rho(u_2-v_1),\\(v_2)_t=(v_2)_{xx}+\rho(v_1-v_2)\end{cases}\tag{4}$$

连结(0,1,1,1)和(1,0,0,0)的行波解。

令 $\tilde{u}_1=u_1,\tilde{u}_2=1-u_2,\tilde{v}_1=1-v_1,\tilde{v}_2=1-v_2$,将其代入系统(4)并省略掉"～",可得到如下等价的合作系统,

$$\begin{cases}(u_1)_t=(u_1)_{xx}+u_1(1-a_1-u_1+a_1v_2),\\(u_2)_t=d(J*u_2-u_2)+r(1-u_2)(a_2u_1-u_2),\\(v_1)_t=(v_1)_{xx}+\rho(u_2-v_1),\\(v_2)_t=(v_2)_{xx}+\rho(v_1-v_2)。\end{cases}\tag{5}$$

从而,系统(4)连接(0,1,1,1)和(1,0,0,0)的行波解等价于系统(5)连结(0,0,0,0)和(1,1,1,1)的行波解。系统(5)连结(0,0,0,0)和(1,1,1,1)的行波解是指满足如下条件的系统(5)的解

$$(u_1,u_2,v_1,v_2)(x,t)=(\varphi_1(\xi),\varphi_2(\xi),\varphi_3(\xi),\varphi_4(\xi)),\ \xi=x+ct,$$

$$\begin{cases}\varphi_1''-c\varphi_1'+\varphi_1(1-a_1-\varphi_1+a_1\varphi_4)=0,\\d(J*\varphi_2-\varphi_2)-c\varphi_2'+r(1-\varphi_2)(a_2\varphi_1-\varphi_2)=0,\\\varphi_3''-c\varphi_3'+\rho(\varphi_2-\varphi_3)=0,\\\varphi_4''-c\varphi_4'+\rho(\varphi_3-\varphi_4)=0,\end{cases}\tag{6}$$

$$\lim_{\xi\to-\infty}(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=(0,0,0,0):=\mathbf{0},\ \lim_{\xi\to+\infty}(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=(1,1,1,1):=\mathbf{1}。\tag{7}$$

在下文,不失一般性,系统(5)的行波解是指边值问题(6)—公式(7)的解。

在本文中,我们采用 R^n 中标准序,即对于向量 $\boldsymbol{x}=(x_1,x_2,\cdots,x_n),\boldsymbol{y}=(y_1,y_2,\cdots,y_n)\in R^n$,如果 $x_i\leqslant y_i,i=1,2,\cdots,n$,那么 $\boldsymbol{x}\leqslant\boldsymbol{y}$;如果 $\boldsymbol{x}\leqslant\boldsymbol{y}$,并且存在一个 i 使得 $x_i<y_i$,那么 $\boldsymbol{x}<\boldsymbol{y}$。

1 准备工作

在本节中,我们给出基本的定义与引理。为了定义和验证系统(6)的上下解,令

$$F(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=\big(F_1(\varphi_1,\varphi_2,\varphi_3,\varphi_4),F_2(\varphi_1,\varphi_2,\varphi_3,\varphi_4),F_3(\varphi_1,\varphi_2,\varphi_3,\varphi_4),F_4(\varphi_1,\varphi_2,\varphi_3,\varphi_4)\big)^{\mathrm{T}},$$

其中

$$F_1(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=\varphi_1''-c\varphi_1'+\varphi_1(1-a_1-\varphi_1+a_1\varphi_4),$$
$$F_2(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=d(J*\varphi_2-\varphi_2)-c\varphi_2'+r(1-\varphi_2)(a_2\varphi_1-\varphi_2),$$
$$F_3(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=\varphi_3''-c\varphi_3'+\rho(\varphi_2-\varphi_3),$$
$$F_4(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=\varphi_4''-c\varphi_4'+\rho(\varphi_3-\varphi_4)。$$

现在,我们给出系统(6)上下解的定义。

定义1 如果向量函数 $\bar{\varphi}(\xi)=\big(\bar{\varphi}_1(\xi),\bar{\varphi}_2(\xi),\bar{\varphi}_3(\xi),\bar{\varphi}_4(\xi)\big)$ 在 R 上连续,除有限点 ξ_i 外,$1\leqslant i\leqslant k$,满足

$$F(\bar{\varphi})\leqslant 0,\ \forall\xi\neq\xi_i,\ 1\leqslant i\leqslant k,$$

以及 $\bar{\varphi}_j'(\xi_i^+)\leqslant\bar{\varphi}_j'(\xi_i^-),1\leqslant i\leqslant k,j=1,2,3,4$,则称 $\bar{\varphi}(\xi)$ 为系统(6)的上解。改变不等号的方向,可以定义系统(6)的下解 $\underline{\varphi}(\xi)=\Big(\underline{\varphi}_1(\xi),\underline{\varphi}_2(\xi),\underline{\varphi}_3(\xi),\underline{\varphi}_4(\xi)\Big)$。

将系统(6)在0处和1处分别线性化,对应的特征矩阵为

$$J_0=\begin{pmatrix}\Gamma_{01}(\lambda) & 0 & 0 & 0\\ ra_2 & \Gamma_{02}(\lambda) & 0 & 0\\ 0 & \rho & \Gamma_{03}(\lambda) & 0\\ 0 & 0 & \rho & \Gamma_{03}(\lambda)\end{pmatrix},$$

其中

$$\Gamma_{01}(\lambda)=\lambda^2-c\lambda+1-a_1,\ \Gamma_{02}(\lambda)=d(\int_R J(y)\mathrm{e}^{-\lambda y}\mathrm{d}y-1)-c\lambda-r,\ \Gamma_{03}(\lambda)=\lambda^2-c\lambda-\rho$$

及

$$J_1=\begin{pmatrix}\Gamma_{11}(\lambda) & 0 & 0 & a_1\\ 0 & \Gamma_{12}(\lambda) & 0 & 0\\ 0 & \rho & \Gamma_{13}(\lambda) & 0\\ 0 & 0 & \rho & \Gamma_{13}(\lambda)\end{pmatrix},$$

其中

$$\Gamma_{11}(\lambda)=\lambda^2-c\lambda-1,\Gamma_{12}(\lambda)=d(\int_R J(y)\mathrm{e}^{-\lambda y}\mathrm{d}y-1)-c\lambda+r(1-a_2),\Gamma_{13}(\lambda)=\lambda^2-c\lambda-\rho,$$

直接计算可得，$\Gamma_{0i}(\lambda)=0$ 和 $\Gamma_{1i}(\lambda)=0$，$i=1,\ 3$，相应的特征根为

$$\lambda_{01}^{\pm}=\frac{c\pm\sqrt{c^2-4(1-a_1)}}{2},\ \lambda_{03}^{\pm}=\frac{c\pm\sqrt{c^2+4\rho}}{2}(\text{二重}),$$

$$\lambda_{11}^{\pm}=\frac{c\pm\sqrt{c^2+4}}{2},\ \lambda_{13}^{\pm}=\frac{c\pm\sqrt{c^2+4\rho}}{2}(\text{二重})。$$

显然当 $c>c^*=2\sqrt{1-a_1}$ 时，$0<\lambda_{01}^-<\lambda_{01}^+$，不难验证，$\Gamma_{i2}(\lambda)=0$，$i=1,\ 2$，均有两个实正根和实负根，分别记为

$$\lambda_{02}^-<0<\lambda_{02}^+,\ \lambda_{12}^-<0<\lambda_{12}^+。$$

由文献[7-8]可得以下引理成立。

引理1 假设(A_1)和(A_2)成立，$c>c^*$，$\lambda_{01}^-<\min\{\lambda_{02}^+,\ \lambda_{03}^+\}$，对充分小的 $\varepsilon>0$，存在 $x\in(R^+)^4$ 使得

$$J_0(\lambda_{01}+\varepsilon)x<0。$$

引理2 令 $f(s)=p\mathrm{e}^{\alpha s}-q\mathrm{e}^{\beta s}$，其中 $0<\alpha<\beta$（或 $\beta<\alpha<0$），$p,q\in R^+$，则

(1) $f(-\infty)=0,f(+\infty)=-\infty$（或 $f(+\infty)=0,f(-\infty)=-\infty$）；

(2) $f(s)=0$，当且仅当 $s=\dfrac{1}{(\beta-\alpha)}\ln\dfrac{p}{q}$。

此外，$f(s)$ 在 $s^*=\dfrac{1}{(\beta-\alpha)}\ln\dfrac{\alpha p}{\beta q}$ 处取到唯一的最大值。

2 行波解的存在性

在本节中，我们将证明本文的主要定理。首先，令

$$\bar{\varphi}(\xi)=\left(\bar{\varphi}_1(\xi),\bar{\varphi}_2(\xi),\bar{\varphi}_3(\xi),\bar{\varphi}_4(\xi)\right)^{\mathrm{T}}:=\left(\mathbf{1}-\bar{\sigma}(\bar{\gamma}\mathrm{e}^{\bar{\alpha}\xi}-\bar{\eta}\mathrm{e}^{\bar{\beta}\xi})\right),$$

$$\underline{\varphi}(\xi)=\left(\underline{\varphi}_1(\xi),\ \underline{\varphi}_2(\xi),\ \underline{\varphi}_3(\xi),\ \underline{\varphi}_4(\xi)\right)^{\mathrm{T}}:=\underline{\sigma}(\ \underline{\gamma}\,\mathrm{e}^{\underline{\alpha}\xi}-\underline{\eta}\,\mathrm{e}^{\underline{\beta}\xi}),$$

其中

$$\underline{\gamma}=(\underline{\gamma}_1,\ \underline{\gamma}_2,\ \underline{\gamma}_3,\ \underline{\gamma}_4)^{\mathrm{T}},\ \bar{\gamma}=(0,\bar{\gamma}_2,0,0)^{\mathrm{T}},\ \underline{\eta}=(\underline{\eta}_1,\ \underline{\eta}_2,\ \underline{\eta}_3,\ \underline{\eta}_4)^{\mathrm{T}},\ \bar{\eta}=(0,\bar{\eta}_2,0,0)^{\mathrm{T}},$$

$\bar{\gamma}_2,\ \bar{\eta}_2\in R^+$，$\underline{\gamma}_i,\underline{\eta}_i\in R$，$i=1,\ 2,\ 3,\ 4$，$0<\underline{\sigma},\ \bar{\sigma}\in R,\ 0>\bar{\alpha}>\bar{\beta}\in R,\ 0<\underline{\alpha}<\underline{\beta}\in R$ 均为待定系数。

进一步,令

$$\bar{\phi}(\xi)=\min\{\bar{\psi}(\xi),1\},\ \underline{\phi}(\xi)=\min\{\underline{\psi}(\xi),0\}。$$

引理3 假设(A_1)和(A_2)成立,$c>c^*$,$\lambda_{01}^-<\min\{\lambda_{02}^+,\lambda_{03}^+\}$,则$\bar{\varphi}$,$\underline{\varphi}$是系统(6)的上下解。

证明 首先,证明$\bar{\varphi}$是系统(6)的上解。显然,当$\xi\leqslant\xi_1=\dfrac{1}{\bar{\alpha}-\bar{\beta}}\ln\dfrac{\bar{\eta}_2}{\bar{\gamma}_2}$时,$\bar{\varphi}_i(\xi)=1$,$i=1,2,3,4$,易证$F_i(\bar{\varphi}_1,\bar{\varphi}_2,\bar{\varphi}_3,\bar{\varphi}_4)=0$,$i=1,3,4$。由于在$R$上,$\bar{\varphi}_2(\xi)\leqslant 1$,因此,$F_2(\bar{\varphi}_1,\bar{\varphi}_2,\bar{\varphi}_3,\bar{\varphi}_4)\leqslant 0$。当$\xi\geqslant\xi_1=\dfrac{1}{\bar{\alpha}-\bar{\beta}}\ln\dfrac{\bar{\eta}_2}{\bar{\gamma}_2}$时,$\bar{\varphi}_i(\xi)=1$,$i=1,3,4$,$\bar{\varphi}_2(\xi)=\bar{\psi}_2(\xi)$,易证

$$F_i(\bar{\varphi}_1,\bar{\varphi}_2,\bar{\varphi}_3,\bar{\varphi}_4)=0,i=1,4,F_3(\bar{\varphi}_1,\bar{\varphi}_2,\bar{\varphi}_3,\bar{\varphi}_4)=\rho(\bar{\psi}_2-1)\leqslant 0。$$

下证$F_2(\bar{\varphi}_1,\bar{\varphi}_2,\bar{\varphi}_3,\bar{\varphi}_4)\leqslant 0$。取$\bar{\beta}<\bar{\alpha}<\lambda_{12}^-$,则不难验证$0<\Gamma_{12}(\bar{\alpha})<\Gamma_{12}(\bar{\beta})$。从而由引理2可知,当$\xi\geqslant\dfrac{1}{\bar{\alpha}-\bar{\beta}}\ln\dfrac{\Gamma_{02}(\bar{\beta})\bar{\eta}_2}{\Gamma_{02}(\bar{\alpha})\bar{\gamma}_2}$时,

$$\Gamma_{02}(\bar{\alpha})\bar{\gamma}_2e^{\bar{\alpha}\xi}-\Gamma_{02}(\bar{\beta})\bar{\eta}_2e^{\bar{\beta}\xi}>0。\tag{8}$$

直接计算可得

$$\frac{1}{\bar{\alpha}-\bar{\beta}}\ln\frac{\Gamma_{02}(\bar{\beta})\bar{\eta}_2}{\Gamma_{02}(\bar{\alpha})\bar{\gamma}_2}<\frac{1}{\bar{\alpha}-\bar{\beta}}\ln\frac{\bar{\eta}_2}{\bar{\gamma}_2}=\xi_1。$$

因此,当$\xi\geqslant\xi_1$时,(8)式成立。由于在R上,$\bar{\varphi}_2(\xi)\leqslant\bar{\psi}_2(\xi)$,所以,当$\xi\geqslant\xi_1$时,对充分小的$\bar{\sigma}>0$,

$$F_2(\bar{\psi}_1,\bar{\psi}_2,\bar{\psi}_3,\bar{\psi}_4)\leqslant-\bar{\sigma}(\Gamma_{12}(\bar{\alpha})\bar{\gamma}_2e^{\bar{\alpha}\xi}-\Gamma_{12}(\bar{\beta})\bar{\eta}_2e^{\bar{\beta}\xi})+r\sigma^2(\bar{\gamma}_2e^{\alpha\xi}-\bar{\eta}_2e^{\beta\xi})^2\leqslant 0。$$

因此,当$\xi\geqslant\xi_1$时,对充分小的$\bar{\sigma}>0$使得

$$F(\bar{\varphi})\leqslant 0。$$

综上所述,对充分小的$\bar{\sigma}>0$,$\bar{\varphi}$是系统(6)的上解。

令$\xi_2=\max\limits_{i=1,2,3,4}\left\{\dfrac{1}{\varepsilon}\ln\dfrac{\underline{\gamma}_i}{\underline{\eta}_i}\right\}$,当$\xi\leqslant\xi_2$时,$\underline{\varphi}(\xi)=\underline{\psi}(\xi)$,并且在$R$上,$\underline{\varphi}(\xi)\geqslant\underline{\psi}(\xi)$。因此,当$\xi\leqslant\xi_2$时,

$$F(\underline{\varphi})\geqslant\underline{\sigma}\begin{pmatrix}\Gamma_{01}(\underline{\alpha})\underline{\gamma}_1e^{\alpha\xi}\\ \Gamma_{02}(\underline{\alpha})\underline{\gamma}_2e^{\alpha\xi}\\ \Gamma_{03}(\underline{\alpha})\underline{\gamma}_3e^{\alpha\xi}\\ \Gamma_{03}(\underline{\alpha})\underline{\gamma}_4e^{\alpha\xi}\end{pmatrix}-\underline{\sigma}\begin{pmatrix}\Gamma_{01}(\underline{\beta})\underline{\gamma}_1e^{\beta\xi}\\ \Gamma_{02}(\underline{\beta})\underline{\gamma}_2e^{\beta\xi}\\ \Gamma_{03}(\underline{\beta})\underline{\gamma}_3e^{\beta\xi}\\ \Gamma_{03}(\underline{\beta})\underline{\gamma}_4e^{\beta\xi}\end{pmatrix}+\begin{pmatrix}0\\ ra_2\underline{\psi}_1\\ \rho\underline{\psi}_2\\ \rho\underline{\psi}_3\end{pmatrix}+\begin{pmatrix}-\underline{\psi}_1^2+a_1\underline{\psi}_1\underline{\psi}_4\\ -ra_2\underline{\psi}_1\underline{\psi}_2+r\underline{\psi}_2^2\\ 0\\ 0\end{pmatrix}=$$

$$\underline{\sigma}\Big(J_0(\alpha)\underline{\gamma}e^{\alpha\xi}-J_0(\underline{\beta})\underline{\eta}e^{\underline{\eta}\xi}\Big)+\begin{pmatrix}-\underline{\psi}_1^2+a_1\underline{\psi}_1\underline{\psi}_4\\ -ra_2\underline{\psi}_1\underline{\psi}_2+r\underline{\psi}_2^2\\ 0\\ 0\end{pmatrix}。$$

由于$c>c^*$且$\lambda_{01}^-<\min\{\lambda_{02}^+,\lambda_{03}^+\}$,取$\underline{\alpha}=\lambda_{01}^-$,$\underline{\beta}=\lambda_{01}+\varepsilon$其中$\varepsilon$充分小,使得

$$\Gamma_{01}(\underline{\alpha})=0,\quad\Gamma_{02}(\underline{\alpha}),\quad\Gamma_{03}(\underline{\alpha})<0。$$

取$\underline{\gamma}_1=1$,$\underline{\gamma}_2=\dfrac{ra_2}{-\Gamma_{02}(\underline{\alpha})}$,$\underline{\gamma}_3=\dfrac{ra_2\rho}{\Gamma_{02}(\underline{\alpha})\Gamma_{03}(\underline{\alpha})}$,$\underline{\gamma}_4=\dfrac{ra_2\rho^2}{-\Gamma_{02}(\underline{\alpha})\Gamma_{03}^2(\underline{\alpha})}$,不难验证

$$J_0(\underline{\alpha})\underline{\gamma}=0。$$

进一步,由引理1,存在$\underline{\eta}\in(R^+)^4$使得

$$J_0(\underline{\beta})\underline{\eta}<0。$$

不难验证,

$$\lim_{\xi\to-\infty}\underline{\sigma}\left(\underline{\gamma}_i e^{\alpha\xi}-\underline{\eta}_i e^{\beta\xi}\right)e^{(\alpha-\beta)\xi}=0,\ \lim_{\xi\to-\infty}\underline{\sigma}\left(\underline{\gamma}_i e^{\alpha\xi}-\underline{\eta}_i e^{\beta\xi}\right)e^{-\alpha\xi}=\underline{\sigma}\underline{\gamma}_i,\ i=1,\ 2,\ 3,\ 4。$$

故在$(-\infty,\xi_2]$上,$\underline{\sigma}\left(\underline{\gamma}_i e^{\alpha\xi}-\underline{\eta}_i e^{\beta\xi}\right)e^{(\alpha-\beta)\xi}$和$\underline{\sigma}\left(\underline{\gamma}_i e^{\alpha\xi}-\underline{\eta}_i e^{\beta\xi}\right)e^{-\alpha\xi}$有界。

因此,当$\xi\leqslant\xi_2$时,对充分小的$\underline{\sigma}>0$,

$$F(\underline{\varphi})\geqslant\underline{\sigma}e^{\beta\xi}\left(-J_0(\underline{\beta})\underline{\eta}\right)+\underline{\sigma}^2e^{\beta\xi}\begin{pmatrix}-e^{(\alpha-\beta)\xi}\underline{\psi}_1e^{-\alpha}\underline{\psi}_1+a_2e^{(\alpha-\beta)\xi}\underline{\psi}_1e^{-\alpha\xi}\underline{\psi}_4\\ a_2e^{(\alpha-\beta)\xi}\underline{\psi}_2e^{-\alpha\xi}\underline{\psi}_2-a_2\gamma e^{(\alpha-\beta)\xi}\underline{\psi}_1e^{-\alpha\xi}\underline{\psi}_2\\ 0\\ 0\end{pmatrix}=$$

$$\underline{\sigma}e^{\beta\xi}\left[\left(-J_0(\underline{\beta})\underline{\eta}\right)+O(\underline{\sigma})\right]\geqslant0。$$

不难验证,当$\xi\geqslant\xi_2$时,$F(\underline{\varphi})\geqslant0$。综上所述,对充分小的$\underline{\sigma}>0$,$\underline{\varphi}$是系统(6)的下解。

取$\bar{\eta}_2>\bar{\gamma}_2$, $\underline{\gamma}_i<\underline{\eta}_i$, $i=1,\ 2,\ 3,\ 4$。从而$\xi_1>0>\xi_2$,因此,由引理3,不难验证,$\bar{\varphi}\geqslant\underline{\varphi}$。最后,利用上下解和不动点定理并结合文献[7-10]的证明过程可得本文的主要结论。

定理1 假设(A_1)和(A_2)成立,$c>c^*$,$\lambda_{01}^-<\min\{\lambda_{02}^+,\ \lambda_{03}^+\}$,边值问题(6)—公式(7)的解存在,即系统(5)存在连结0和1的行波解。

注1 本文采用文献[7]中的思想,利用核函数的性质,将系统(1)转化为四个方程耦合而成的系统(4),再转化成对应的合作系统(5),最后构造上下解得到结论,将文献[8]中的结果推广到强核上。

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