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随机延迟容忍的变步长自适应滤波

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摘　要:在实际的自适应滤波系统中普遍存在随机处理延迟和异质测量噪声(如高斯噪声、脉冲噪声等)的问题,而现有的变步长最小均方误差(Variable Step-Size Least Mean Square,VSSLMS)算法在分析时通常假设系统为无延时系统. 为了解决上述问题,提出一种随机延迟容忍的鲁棒VSSLMS算法,利用Squareplus函数的两个优势:(1)在时延条件下对梯度估计稳定性具有固有平滑性;(2)针对多种类型分布的非线性干扰具有抑制能力. 在理论上分析该算法的均方误差和稳态均方误差以评估其性能,并设计系统辨识实验仿真来验证该算法的有效性,且结果与理论分析一致,也优于现有的自适应滤波算法. 因此提出的算法不仅表现出更好的稳态性能,在对抗随机时延和多类型测量噪声时也具有更好的鲁棒性.

关键词:随机时延,变步长,Squareplus函数,各类干扰
中图分类号:TN911.72　　**文献标志码**:A

Stochastic delay-tolerant adaptive filtering via variable step-size

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Abstract: In practical adaptive filtering systems, stochastic processing delays and heterogeneous measurement noises, such as Gaussian noise and impulsive noise, are commonly encountered. However, existing variable step-size least mean square (VSSLMS) algorithms typically assume a delay-free system in analysis. To address this limitation, we propose a stochastic delay-tolerant robust VSSLMS algorithm. The proposed method leverages two key advantages of the Squareplus function. Firstly, it's inherently smooth, which stabilizes gradient estimation under time-delayed conditions. Secondly, it's capable to suppress nonlinear interference arising from multiple types of noise distributions. We theoretically analyze the algorithm's mean square error (*MSE*) and steady-state *MSE* to evaluate its performance. Furthermore, system identification experiments are conducted via simulation to verify the effectiveness of the proposed algorithm. The experimental results align well with the theoretical analysis and demonstrate superior performance compared to existing adaptive filtering algorithms. Consequently, the proposed algorithm not only achieves better steady-state performance but also exhibits enhanced robustness in the presence of stochastic time delays and diverse types of measurement noises.

Keywords: random time-delay, variable step-size, Squareplus function, various noises

通信网络系统中的时延[1-4]可分为两类[5]:通信时延[6-10]和处理时延[11],针对后者已提出了多种时延估计方法[12-15]. 最小均方误差(Least Mean Square,LMS)算法因其独特的优势被广泛应用于自适应滤波. 首先,LMS算法在源信号是带限且平滑的前提下,不依赖任何先验的信号统

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计信息或频谱特性,减少了对信号先验知识的需求. 其次,此类算法能在潜在的非平稳环境中有效跟踪时延,例如传感器移动导致的时延变化. 在时延估计任务中,自适应滤波的常见实现方式是假设滤波器系数是sinc函数的一个样本[16]. 为了计算滤波器系数对应的时延,此类算法通常采用插值方法[17]或查找表[12]来提高估计精度.

近年来,尽管已有大量关于时延估计的研究成果被提出,但其中大多数研究都基于时延是一个固定值的假设,然而在实际应用中,时延数据处理设备往往受到工作环境(如压力、温度)以及待处理信号(如频率、电压)的影响. 因此,本文的主要研究内容是存在随机时延时实现鲁棒的自适应估计. 此外,非高斯测量噪声在实际工程中普遍存在,并对估计精度产生了严重的影响. 当测量噪声中包含非高斯噪声时,以最小均方误差准则为基础的自适应滤波算法可能会面临严重的收敛性能下降,甚至导致发散问题[18−39],测量噪声和时延会影响估计误差和迭代学习效果. 变步长(Variable Step Size, VSS)是一种可以有效解决迭代学习中稳态误差和收敛速度之间的矛盾的方法. 近年来,针对这一问题,已有多种VSSLMS算法[38,40−50]被提出. Dariusz et al[51]对几种最常见的变步长归一化最小均方误差(Variable Step-Size Normalized Least Mean Square, VSSNLMS)算法进行总结并对比了它们的性能,发现没有一种VSSNLMS算法能适用所有场景,因为不同的应用环境对自适应算法提出了不同的挑战,因此需要设计不同的步长更新策略,以实现最优性能. Bin Saeed et al[52]提出一种带有增量方案的VSS策略,而Bin and Zerguine[43]首次系统讨论了变步长增量最小均方误差(Variable Step-Size Incremental Least Mean Square, VSSILMS)算法,这种方法在高信噪比的场景下表现良好.

VSSLMS算法时常忽略随机时延,因此,本文针对数据处理时延(即系统的自适应分支在传感和输入信号准备过程中存在时延)与各类测量噪声同时存在的情况,提出一种受Squareplus函数启发的鲁棒VSSLMS算法. 下文中,黑体表示向量,$[\cdot]^{\mathrm{T}}$表示转置,$[\cdot]^{-1}$表示逆矩阵运算.

1 VSSLMS算法的提出

考虑一个系统识别问题,期望信号表示为:

$$d(n)=\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)+\rho(n) \tag{1}$$

其中,$\rho(n)$为加性噪声,$\rho(n)$表示均值为0、方差为σ_ρ^2且满足平稳分布的随机序列. 此外,其奇数阶矩为零,并且假定与其他信号无关. $\boldsymbol{W}_0\in R^{L\times 1}$为$L$阶的未知系统参数矢量. $\boldsymbol{X}(n)$表示均值为0、方差为σ_x^2的平稳分布高斯噪声,其自相关矩阵是正定矩阵$\boldsymbol{R}_{XX}(0)=E\left[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n)\right]$.

1.1 LMS算法 系统估计误差信号可表示为:

$$e(n)=d(n)-y(n) \tag{2}$$

滤波器的输出信号为:

$$y(n)=\boldsymbol{W}^{\mathrm{T}}(n)\boldsymbol{X}(n) \tag{3}$$

假设:

$$\boldsymbol{V}(n)=\boldsymbol{W}(n)-\boldsymbol{W}_0 \tag{4}$$

其中,$\boldsymbol{W}_0$表示长度为L的未知系统的权向量. 故:

$$\begin{aligned}e(n)&=\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)+\rho(n)-\boldsymbol{W}^{\mathrm{T}}(n)\boldsymbol{X}(n)=\\&-\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}(n)+\rho(n)\end{aligned} \tag{5}$$

为了寻找最优解,已有各种基于误差信号$e(n)$的误差优化准则被提出,其中一种用于自适应滤波算法的代价函数为:

$$\boldsymbol{W}(n)=\operatorname{argmin}J\big(\boldsymbol{W}(n)\big)=\operatorname{argmin}\left[e^2(n)\right] \tag{6}$$

优化算法的梯度项表示为:

$$\frac{\partial J\big(\boldsymbol{W}(n)\big)}{\partial\boldsymbol{W}(n)}=\frac{\partial}{\partial\boldsymbol{W}(n)}\big(e^2(n)\big)=-2e(n)\boldsymbol{X}(n) \tag{7}$$

迭代方案如下:

$$\begin{aligned}\boldsymbol{W}(n+1)&=\boldsymbol{W}(n)-\mu\frac{\partial J\big(\boldsymbol{W}(n)\big)}{\partial\boldsymbol{W}(n)}=\\&\boldsymbol{W}(n)+2\mu e(n)\boldsymbol{X}(n)\end{aligned} \tag{8}$$

其中,μ为步长.

1.2 随机信号处理时延 期望信号中的随机信号处理时延[11]可作为自适应滤波通信网络节点的参考. 在连接的通信链路中,时延始终存在,因此假设时延$\tau(n)$为一个随机变量. 然而,过量的时延可能导致通信中断,因此必须限制最大时延[7−9,53]. 假设存在$B\geqslant 1$使$\forall n>0,\tau(n)\leqslant B$并且$Prob\big(\tau(n)=l\big)=p_l, l=0,1,2,\cdots,B,\forall n>0$. 由于数据处理设备的时延受到工作环境(如压力、温度)和待处理信号(如频率、电压)的影响[54],因

此,进一步假设时延$[0,1,2,\cdots,B]$服从参数为λ的泊松分布:$\tau(n)\sim\pi(\lambda)$,其概率密度函数为:

$$pdf\left[\tau(n)\right]=\frac{\lambda^n e^{-\lambda}}{n!} \tag{9}$$

1.3 Squareplus函数 式(10)所示的Squareplus函数由一个超参数$b\geqslant 0$定义,该参数决定函数在$x=0$附近弯曲区域的"平滑度":

$$\text{squareplus}\left(e(n)\right)=\frac{1}{2}\left(e(n)+\sqrt{e^2(n)+b}\right) \tag{10}$$

其一阶导数为:

$$\frac{\mathrm{d}}{\mathrm{d}\left(e(n)\right)}\text{squareplus}\left(e(n)\right)=\frac{1}{2}\left(1+\frac{e(n)}{\sqrt{e^2(n)+b}}\right) \tag{11}$$

与Squareplus函数本身类似,其一阶导数具有代数形式,并且计算效率较高.Squareplus函数的导数可视作一种"代数Sigmoid"函数$\frac{x}{\sqrt{x^2+1}}$(经过适当缩放和相应平移).与Softplus函数类似,对于所有有效参数b,Squareplus函数在原点的一阶导数值均为0.5:

$$\forall b\geqslant 0$$
$$\frac{\mathrm{d}}{\mathrm{d}\left(e(n)\right)}\text{squareplus}\left(e(n)=0\right)=\frac{1}{2}\text{softplus}=\frac{1}{2} \tag{12}$$

图1为当b取不同值时,Squareplus函数(及其一阶、二阶导数)沿Softplus函数的表现.

本文提出一种基于增量方案的适用于低信噪比场景的VSS策略.受Barron[55]的启发,提出了步长更新公式,其更新机制由递归控制关系描述,如式(13)所示:

$$\mu(n)=\frac{a}{2}\frac{e^2(n)}{\sqrt{e^4(n)+b}} \tag{13}$$

其中,$0<a\leqslant 1$,决定算法在非稳态阶段的步长峰值,直接控制算法的收敛速度.将式(13)代入式(8),得到VSSLMS算法的步长变量$\mu(n)$.图2展示了基于改进的Squareplus函数,变量a和b取不同值时$\mu(n)$的表现.

针对参数a和b的具体选择策略,其值可以通过构建稳定时的*MSE*(Mean Squared Error)或*MSD*(Steady-State Mean-Square Deviation)与a,b的关系式,进而通过此关系式求得最优解,但这将显著增加系统的计算复杂度.考虑到本文的研究重点在于解决通信网络中存在的随机时延干扰,且目标是在低信噪比环境下保持算法的鲁棒性与低复杂度特性,因此本文不再对a和b引入额外的自适应迭代环节.采用经典的实验对比法,根据不同取值下算法收敛速度与稳态误差的综合表现,选取一组固定的最优参数值,这种处理方式在有效验证抗时延性能的同时,能够最大程度降低算法的计算负担.

1.4 随机时延VSSLMS算法的提出 当$\tau(n)>0$时误差信号受随机信号处理时延的影响,系统估计误差信号可表示为:

$$e(n)=\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)+\rho(n)-\boldsymbol{W}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right) \tag{14}$$

因此,VSSLMS算法的权重更新为:

$$\boldsymbol{W}(n+1)=\boldsymbol{W}(n)+2\mu(n)e(n)\boldsymbol{X}\left(n-\tau(n)\right)=\boldsymbol{W}(n)+\left(\frac{ae^2(n)}{\sqrt{e^4(n)+b}}\right)e(n)\boldsymbol{X}\left(n-\tau(n)\right) \tag{15}$$

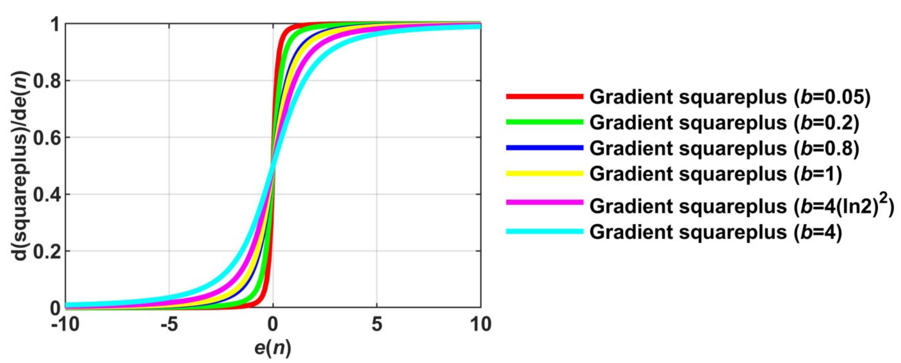


图1 参数b取不同值Squareplus函数及其一阶导数的表现

Fig. 1 A visualization of Squareplus with different values of the b hyperparameter, as well as the first deri-vative

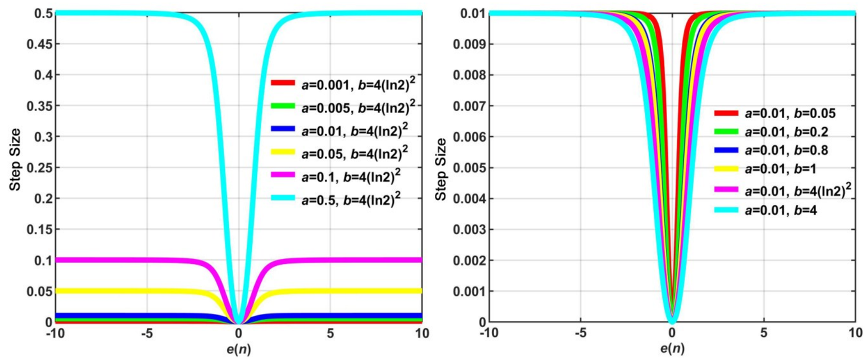


图 2 改进 Squareplus 函数下变量 a 和 b 取不同值时 $\mu(n)$ 的更新情况

Fig.2 A visualization of $\mu(n)$ based on a modified Squareplus for different values of a and b

当 $\tau(n)=0$ 时,式(15)可简化为 VSSLMS 算法的标准形式. 需要注意的是,式(14)和文献[11]中的问题是一致的(尤其以式(2)为例),但本文的区别在于引入随机时延的影响.

2 VSSLMS 算法的性能

进一步分析所提自适应滤波算法的性能,包括均方误差(MSE)、稳态均方误差(MSD)$\left(MSD(n)=E\left[V^{\mathrm{T}}(n)V(n)\right]\right)$以及计算复杂度. 理论推导基于文献[9,11,56-60]中的相关假设,如下所述. (1)时间 $m\geqslant n$ 时,时间 n 的权重与输入矢量 $\boldsymbol{X}(n)$ 统计独立. (2)过量时延可能导致节点脱离通信链路系统,需要限制最大时延,即当 $\lim\limits_{n\to\infty}\boldsymbol{X}\left(n-\tau(n)\right)$ 是一个平稳的、均值为零的、独立同分布的高斯随机变量序列,且其方差为 σ_x^2、正定协方差矩阵为 $\boldsymbol{R}_{XX}(0)=E\left[\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\right]$ 时有 $\forall n>0,\tau(n)\leqslant B$. (3)噪声 $\rho(n)$ 与输入信号 $\boldsymbol{X}(n)$ 和 $\boldsymbol{X}\left(n-\tau(n)\right)$ 相互独立.

(4)回归向量 $\boldsymbol{X}(n)$, $\boldsymbol{X}\left(n-\tau(n)\right)$ 与 $\boldsymbol{V}(n)$ 相互独立.

2.1 均方误差 由式(4)、式(14)和式(15)可得:

$$\begin{aligned}
\boldsymbol{V}(n+1)=&\boldsymbol{V}(n)+2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\left[\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0+\rho(n)-\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}(n)\right]=\\
&\boldsymbol{V}(n)-2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\\
&2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0+2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\rho(n)=\\
&\left[\boldsymbol{I}-2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\right]\boldsymbol{V}(n)+\\
&2\mu(n)\left[\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)-\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\right]\boldsymbol{W}_0+2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\rho(n)
\end{aligned}\tag{16}$$

假设显著简化自适应滤波的随机过程分析,因此,基于假设(1)~(4),对式(16)的两边求整体均值:

$$\begin{aligned}
E\left[\boldsymbol{V}(n+1)\right]=&E\left[\left(\boldsymbol{I}-2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\right)\boldsymbol{V}(n)\right]+\\
&E\left[2\mu(n)\left(\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)-\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\right)\boldsymbol{W}_0\right]+E\left[2\mu(n)\boldsymbol{X}\left(n-\tau(n)\right)\rho(n)\right]=\\
&\left(\boldsymbol{I}-2E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)\right)E\left[\boldsymbol{V}(n)\right]+2E\left[\mu(n)\right]\left\{\boldsymbol{R}_{XX}\left(\tau(n)\right)-\boldsymbol{R}_{XX}(0)\right\}\boldsymbol{W}_0+\\
&2E\left[\mu(n)\right]E\left[\boldsymbol{X}\left(n-\tau(n)\right)\right]E\left[\rho(n)\right]=\\
&\left(\boldsymbol{I}-2E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)\right)E\left[\boldsymbol{V}(n)\right]+2E\left[\mu(n)\right]\left\{\boldsymbol{R}_{XX}\left(\tau(n)\right)-\boldsymbol{R}_{XX}(0)\right\}\boldsymbol{W}_0
\end{aligned}\tag{17}$$

通过对式(13)求期望平均以确定 $\mu(n)$ 的统计特征,如式(18)所示:

$$E\left[\mu(n)\right]=E\left[\frac{ae^2(n)}{\sqrt{e^4(n)+b}}\right]\tag{18}$$

其中,若$\left|e(n)\right| \ll b$,则$E\left[\dfrac{ae^2(n)}{\sqrt{e^4(n)+b}}\right]=0$,若$\left|e(n)\right| \gg b$,则$E\left[\dfrac{ae^2(n)}{\sqrt{e^4(n)+b}}\right]=a$.

由式(18)可知,VSSLMS算法的收敛因子必须在以下范围内选取,如式(19)所示:

$$0<E\left[\mu(n)\right]<\frac{1}{\lambda_{\max}} \tag{19}$$

其中,$\lambda_{\max}=\max\left\{\lambda_i \mid i=1,2,\cdots,L\right\}$. 当$\lambda_{\max}<\mathrm{Tr}\left\{\boldsymbol{R}_{XX}(0)\right\}$时,所提自适应滤波算法的均值收敛有一个更严格直接的限制性条件:

$$0<E\left[\mu(n)\right]<\frac{1}{\mathrm{Tr}\left\{\boldsymbol{R}_{XX}(0)\right\}} \tag{20}$$

其中,$\mathrm{Tr}\left\{\boldsymbol{R}_{XX}(0)\right\}$表示矩阵$\boldsymbol{R}_{XX}(0)$的迹.

基于式(20),$\boldsymbol{E}\left[\boldsymbol{V}(n+1)\right]=\boldsymbol{E}\left[\boldsymbol{V}(n)\right]$恒成立,有$\boldsymbol{E}\left[\boldsymbol{V}(\infty)\right]=\lim\limits_{n\to\infty}\boldsymbol{E}\left[\boldsymbol{V}(\infty)\right]=2E\left[\mu(\infty)\right]\left[\boldsymbol{R}_{XX}(\tau(\infty))-\boldsymbol{R}_{XX}(0)\right]\boldsymbol{W}_0=0$,即$\boldsymbol{E}\left[\boldsymbol{W}(\infty)\right]=\boldsymbol{W}_0$,因此,即使在由于数据处理而产生确定范围的时延时,所提的VSSLMS算法也能用于有效的参数估计.

2.2　稳态性能　对式(16)左乘其转置,并求期望值,得:

$$
\begin{aligned}
&MSD(n+1)=E\left[\boldsymbol{V}^{\mathrm{T}}(n+1)\boldsymbol{V}(n+1)\right]=\\
&E\left[\begin{pmatrix}\left[\boldsymbol{I}-2\mu(n)\boldsymbol{X}(n-\tau(n))\right]\boldsymbol{V}(n)+\\ 2\mu(n)\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n)-\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]\boldsymbol{W}_0+\\ 2\mu(n)\boldsymbol{X}(n-\tau(n))\rho(n)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\left[\boldsymbol{I}-2\mu(n)\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]\boldsymbol{V}(n)+\\ 2\mu(n)\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n)-\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]\boldsymbol{W}_0+\\ 2\mu(n)\boldsymbol{X}(n-\tau(n))\rho(n)\end{pmatrix}\right]=\\
&E\left[\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{V}(n)\right]+4E\left[\mu^2(n)\right]E\left[\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{V}(n)\right]-\\
&4E\left[\mu(n)\right]E\left[\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{V}(n)\right]-\\
&4E\left[\boldsymbol{V}^{\mathrm{T}}(n)\right]\begin{pmatrix}E\left[\mu(n)\right]E\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n)\right]+E\left[\mu(n)\right]E\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]+\\ E\left[\mu^2(n)\right]E\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]+\\ E\left[\mu^2(n)\right]E\left(\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right)\end{pmatrix}+\\
&4E\left[\mu^2(n)\right]\boldsymbol{W}_0^{\mathrm{T}}\begin{pmatrix}E\left[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n)\right]-\\ 2E\left[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]+\\ E\left[\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]\end{pmatrix}\boldsymbol{W}_0+\\
&4E\left[\mu^2(n)\right]E\left[\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\right]\sigma_\rho^2=\\
&\left(1-4E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)+4E\left[\mu(n)\right]^2\boldsymbol{R}_{XX}^2(0)\right)MSD(n)-\\
&4E\left[\boldsymbol{V}^{\mathrm{T}}(n)\right]\begin{pmatrix}E\left[\mu(n)\right]\boldsymbol{R}_{XX}(\tau(n))+E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)-2E\left[\mu^2(n)\right]\boldsymbol{R}_{XX}^2(0)+\\ 2E\left[\mu^2(n)\right]E\left[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]\end{pmatrix}+\\
&4E\left[\mu^2(n)\right]\boldsymbol{W}_0^{\mathrm{T}}\left(\boldsymbol{R}_{XX}^2(\tau(n))-2E\left[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\boldsymbol{X}(n-\tau(n))\boldsymbol{X}^{\mathrm{T}}(n-\tau(n))\right]+\boldsymbol{R}_{XX}^2(0)\right)\boldsymbol{W}_0+\\
&4E\left[\mu^2(n)\right]\boldsymbol{R}_{XX}(0)\sigma_\rho^2
\end{aligned} \tag{21}
$$

假设式(21)收敛，需满足 $\frac{4E\left[\mu^2(n)\right]\boldsymbol{R}_{XX}^2(0)}{4E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)}=E\left[\mu(n)\right]\boldsymbol{R}_{XX}(0)<1\rightarrow E\left[\mu(n)\right]<\frac{1}{\boldsymbol{R}_{XX}(0)}$ 符合式(19)和式(20). 此外，基于上述分析，稳态 $\left|e(n)\right|\ll b, \boldsymbol{E}\left[\mu(n)\right]=\boldsymbol{E}\left[\mu^2(n)\right]=0$ 且 $\boldsymbol{E}\left[\boldsymbol{V}(\infty)\right]=\lim_{n\rightarrow\infty}\boldsymbol{E}\left[\boldsymbol{V}(\infty)\right]=0$.

稳态条件下，$MSD(\infty)=\lim_{n\rightarrow\infty} MSD(n)=0$，表明即使存在由数据处理引起的、具有确定范围的时延，VSSLMS算法仍能有效地将权重从初始值调整至最优值.

时刻 n 的均方误差由式(22)给出：

$$
\begin{aligned}
&MSE(n)=\boldsymbol{E}\left[e^2(n)\right]=\\
&\boldsymbol{E}\begin{bmatrix}\left(\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)-\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}\left(n-\tau(n)\right)-\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)+\rho(n)\right)\cdot\\ \left(\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0-\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+\rho(n)\right)\end{bmatrix}=\\
&E\begin{bmatrix}\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\boldsymbol{W}_0^{\mathrm{T}}X\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)W_0-\boldsymbol{V}^{\mathrm{T}}(n)X\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0+\\
\rho(n)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0+\\
\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0+\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0-\\
\rho(n)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0-\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+\\
\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)-\\
\rho(n)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)\rho(n)-\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}\left(n-\tau(n)\right)\rho(n)-\\
\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\rho(n)+\rho(n)\rho(n)\end{bmatrix}=\\
&E\begin{bmatrix}\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0-\\
2\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}(n)\boldsymbol{W}_0+2\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{W}_0+\\
\boldsymbol{V}^{\mathrm{T}}(n)\boldsymbol{X}\left(n-\tau(n)\right)\boldsymbol{X}^{\mathrm{T}}\left(n-\tau(n)\right)\boldsymbol{V}(n)+\rho(n)\rho(n)\end{bmatrix}=\\
&2\left(\boldsymbol{R}_{XX}(0)-\boldsymbol{R}_{XX}\left(\tau(n)\right)\right)\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{W}_0+2\left(\boldsymbol{R}_{XX}(0)-\boldsymbol{R}_{XX}\left(\tau(n)\right)\right)\boldsymbol{W}_0^{\mathrm{T}}V(n)+\boldsymbol{R}_{XX}(0)MSD(n)+\sigma_\rho^2
\end{aligned}
\tag{22}
$$

当 $n\rightarrow\infty$ 时，

$$
\begin{aligned}
MSE(\infty)=&2\left(\boldsymbol{R}_{XX}(0)-\boldsymbol{R}_{XX}\left(\tau(\infty)\right)\right)\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{W}_0+\boldsymbol{R}_{XX}(0)MSD(\infty)+\sigma_\rho^2=\\
&2\left(\boldsymbol{R}_{XX}(0)-\boldsymbol{R}_{XX}\left(\tau(\infty)\right)\right)\boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{W}_0+\sigma_\rho^2\geqslant\sigma_\rho^2
\end{aligned}
\tag{23}
$$

式(21)和式(23)表明，存在数据处理时延时，不论是否采用变步长策略，MSE 都会增加，但若采用变步长策略，MSD 在理论上可以趋近于零.

3 仿真结果

在仿真过程中引入随机时延以进行系统辨识，并利用仿真结果验证VSSLMS算法的鲁棒性及其自适应估计的准确性. 为了全面评估所提算法的性能，与几种常见的变步长策略算法进行对比，包括VSSILMS[43]，MVCLMS[38]，SVSLMS[42]以及VSNLMS[40]算法. 在所有实验中，系数向量初始化为零，仿真条件如下.

(1)自适应滤波器的长度 $L=7$.

(2)未知有限冲激响应系统的权矢量：

$\boldsymbol{W}_0(n)=$

$[0.6, -0.4, 0.25, -0.15, 0.1, -0.05, 0.001]^{\mathrm{T}}$

(3)输入信号 $\boldsymbol{X}(n)$ 是一个均值为零，方差 $\sigma_x^2=1$ 的高斯白噪声序列.

3.1 固定时延和随机时延的理论和仿真对比

对于一个随机生成的未知系统，图3验证了VSSLMS算法的理论和仿真的稳态性能，测量噪声为高斯白噪声. 图3a为VSSLMS算法下的固定时延，$\tau(n)=0, 1, 2, 3, 4, 5$；图3b为VSSLMS算法下的随机时延，其中 $\lambda=1.2, 1.5, 1.8, 2, 2.5, 3$ 满足泊松分布，即MATLAB R2016b中的“poissrnd.m”. 该结果是当信噪比 $SNR=20$ dB时，使用40个独立运行集和1000次迭代经由蒙特

卡罗仿真得到的. 由图可见,仿真结果与式(21)和式(23)的理论结果相一致.

3.2　随机时延强度下的稳态性能　对比输入信号$X(n)$含有高斯分布噪声时的VSSLMS算法和传统LMS算法. 如图4所示,相比于传统的LMS算法,本文提出的VSSLMS算法在处理时延时性

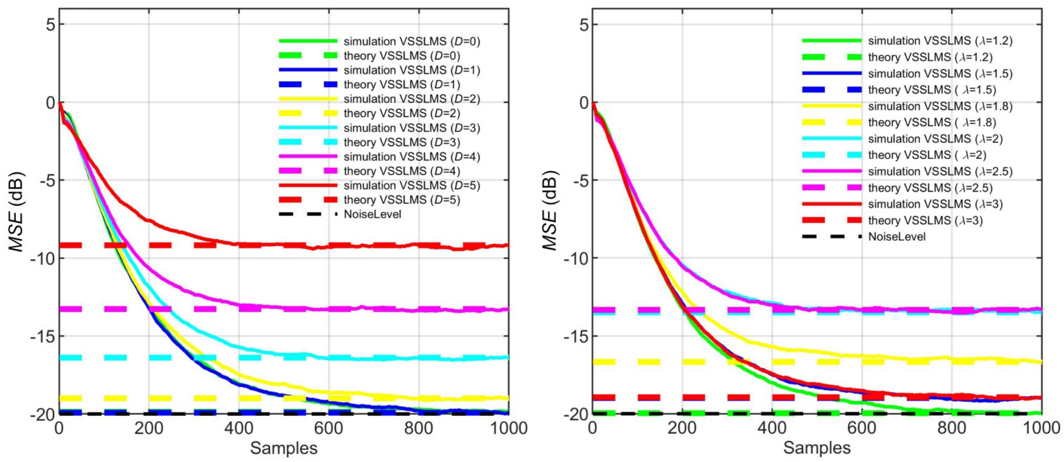


图3　VSSLMS算法的稳态均方误差(*MSE*)的对比

Fig.3　*MSE* of experimental and analytical steady-state

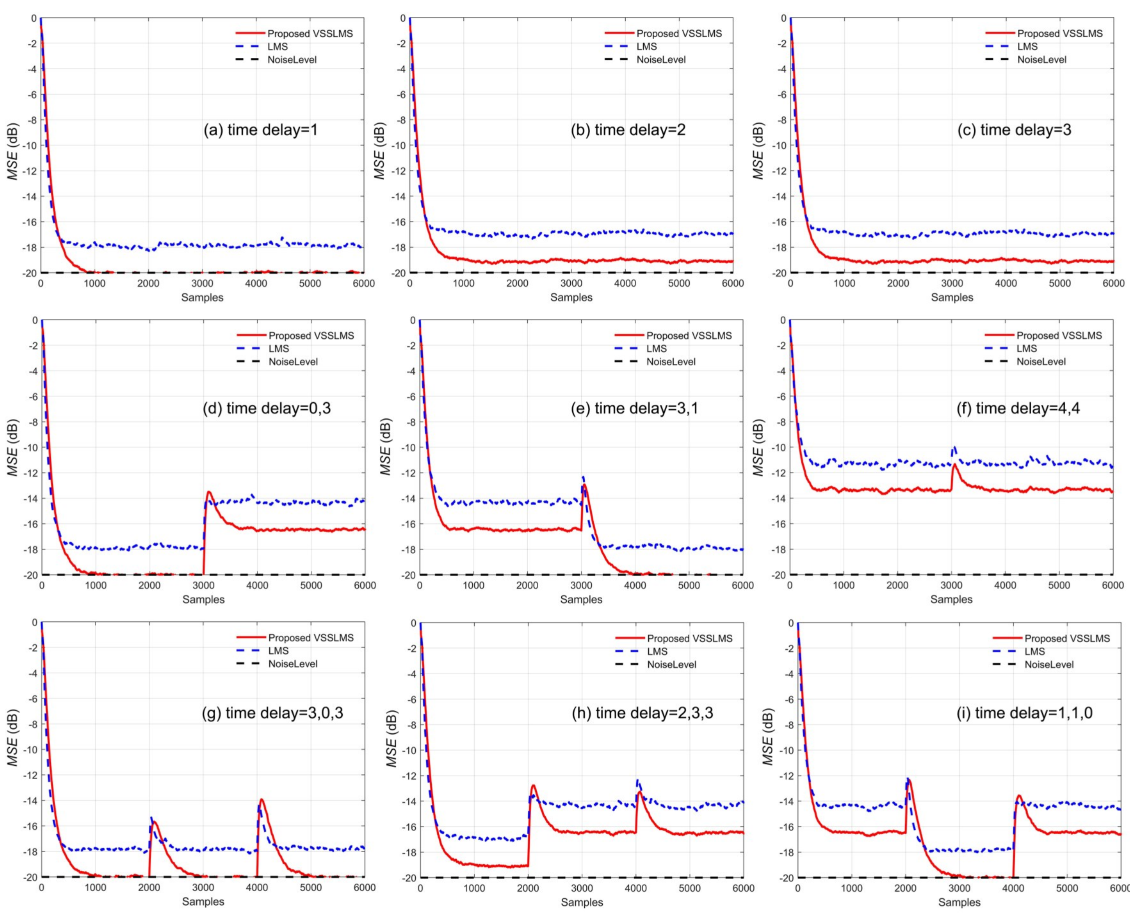


图4　VSSLMS算法的*MSE*跟踪对比

Fig.4　A comparative analysis of the tracking *MSE*

能更好．此外，无论时延是固定的还是频繁变化的，VSSLMS算法都能更快速且准确地识别系统，自适应滤波都在早期收敛阶段有更大的步长．当算法稳定时，步长会迅速减小，使该算法能快速准确地估计系统参数．

LMS算法$(\mu=0.1)$含有高斯白噪声．结果是当信噪比$SNR=20$ dB时，使用40个独立运行集和6000次迭代经由蒙特卡洛仿真得到．图4a～c在时刻3001存在一个时延，图4d～f在时刻1和3001分别存在两个时延，图4g～i在时刻1，2001和4001分别存在三个时延，并且这些时延满足参数$\lambda=1.5$的泊松分布．

3.3　随机时延的稳态性能对比　对测量噪声为高斯白噪声的VSSLMS算法与几个常见VSS算法进行对比：VSSILMS[43]，MVCLMS[38]，SVSLMS[42]以及VSNLMS[40]算法．如图5所示，VSSLMS算法在处理时延时和其他算法相比性能更优．此外，无论时延是固定的还是变化的，VSSLMS算法都能更快速且准确地识别系统，自适应滤波都在早期收敛阶段具有更大的步长．当算法稳定时，步长会迅速减小，使算法能快速且准确地估计系统参数．

VSSLMS算法$(a=0.1,b=4\ln^2 2)$，VSSILMS算法$(\alpha=0.95,\gamma=0.001)$，MVCLMS算法$\left(r=0.003,\tau=\left(\frac{1}{2r}\right)^2\right)$，SVSLMS算法$(\alpha=0.2,\beta=0.1)$和VSNLMS算法$(\alpha=1,\beta=2,c=2)$均含有高斯白噪声．该结果是当信噪比$SNR=20$ dB时，使用40个独立运行集和6000次迭代经由蒙特卡洛仿真得到的．如图5所示，分别在时刻1，2001和4001有三个时延，并且这些时延服从参数$\lambda=1.5$的泊松分布．

3.4　非高斯白噪声下的随机时延的稳态性能对比　对测量噪声为非高斯白噪声的VSSLMS算法与几个常见VSS算法进行对比：VSSILMS[43]，MVCLMS[38]，SVSLMS[42]以及VSNLMS[40]算法．如图6所示，VSSLMS算法在处理时延时比其他算法性能更优．无论时延是固定的还是变化的，即使在测量噪声的分布特性发生变化时，VSSLMS算法都能更快速且准确地识别系统．此外，无论时延是否为固定值，自适应滤波都在早期收敛阶段有更大的步长．当算法稳定时，步长会迅速减小，使算法能够快速且准确地估计系统参数．

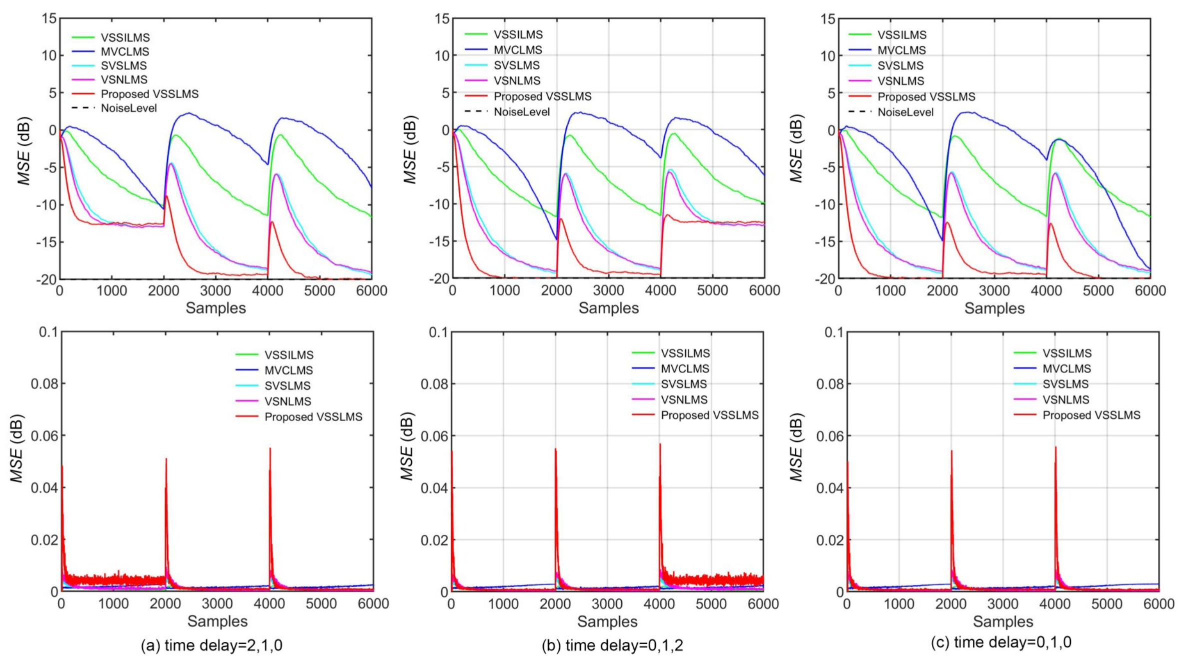
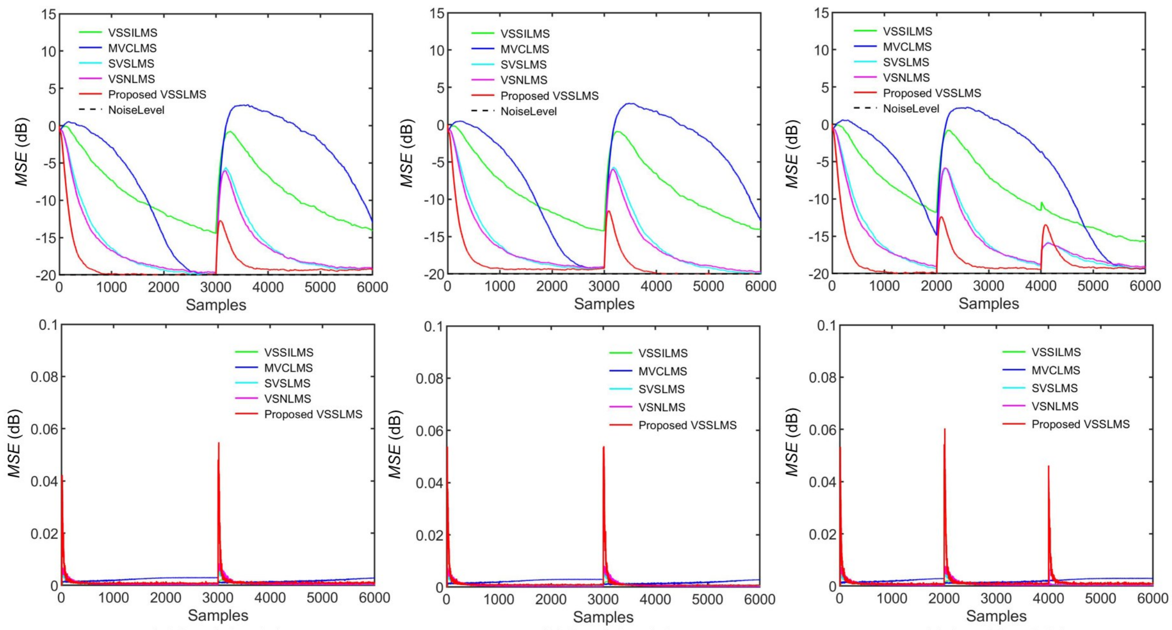


图5　输入信号为$X(n)$时VSSLMS算法的MSE跟踪对比

Fig.5　The tracking MSE of the proposed VSSLMS algorithm with the input signal $X(n)$



(a) time delay=0,1　(b) time delay=1,0　(c) time delay=0,1,1

图6　输入信号为 $X(n)$ 时 VSSLMS 算法的 MSE 跟踪对比

Fig.6　The tracking MSE of the proposed VSSLMS algorithm with the input signal $X(n)$

VSSLMS 算法($a=0.1, b=4\ln^2 2$),VSSILMS 算法($\alpha=0.95, \gamma=0.001$),MVCLMS 算法$\left(r=0.003, \tau=\left(\frac{1}{2r}\right)^2\right)$,SVSLMS 算法($\alpha=0.2, \beta=0.1$)以及 VSNLMS 算法($\alpha=1, \beta=2, c=2$)均含有非高斯白噪声.该结果是当信噪比 $SNR=20$ dB 时,使用40个独立运行集和6000次迭代经由蒙特卡洛仿真得到的.图6a的测量噪声是参数为2的瑞利分布噪声,图6b的测量噪声是均匀分布在(0,1)的噪声,图6c的测量噪声满足参数为2的泊松分布.这些时延都满足参数 $\lambda=1.5$ 的泊松分布.

4　结论

尽管随机时延已被证明会导致梯度对齐和收敛速度的失真,但在实际应用中,仍普遍存在系统性忽略这一因素的情况.针对这一问题,本文提出一种基于 Squareplus 函数的时延感知的鲁棒 VSSLMS 算法.此外,通过均方误差和稳态均方误差评估该算法的性能,并通过系统辨识仿真验证该算法的有效性,结果与理论分析的结果相一致,且优于现有的鲁棒技术.提出的 VSSLMS 算法优于一些 VSS 算法,如 VSSILMS,MVCLMS,SVSLMS 以及 VSNLMS 算法.该算法不仅表现出更好的稳态性能,在对抗随机时延和多类型测量噪声时还具有更好的鲁棒性.

尽管 VSSLMS 算法具有优异的性能,但实际工程应用的环境是复杂和动态的[61-63],需要与不同的应用场景相适应.同时,本研究仍存在一定局限性.在时延建模方面,本文基于通信系统场景采用泊松分布对随机时延进行描述,然而,实际物理系统中的时延分布可能更为复杂多变.未来有必要进一步探讨并验证更多样化、更贴近极端工况的时延模型.

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