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含扰动约束的非线性系统无偏模型预测控制

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摘 要: 文章提出一种约束系统的无偏模型预测控制算法,此方法利用人工干扰来代替建模误差。该算法基于新的扩展系统结构和双控制律实现,其中,双控制律由线性稳定控制和动态预测控制组成。与以往方法相比,该算法不需要计算稳态输入和状态目标,也不需要使用增广状态观测器,具有扩展系统模型可控和模型失配处理等特点。通过考虑状态约束和输入约束,MPC控制器的设计既能达到无偏控制的目的,又能在存在干扰和模型失配的情况下保证约束满足。针对连续非线性夹套搅拌槽式反应器(CSTR)模型进行了仿真,说明了该算法的有效性。

关键词: 过程控制; 无偏移控制; 模型预测控制; 约束系统; 干扰抑制
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An offset-free model predictive control for constrained nonlinear systems with disturbances

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Abstract: This paper proposes an offset-free model predictive control (MPC) algorithm for constrained nonlinear systems, by utilizing an artificial disturbance to replace the modeling error. The algorithm is based on a new expanded system structure and the dual part control law consisting of the linear stabilizing control and the dynamic predictive control. Compared with previous techniques, there is no need for calculating the steady-state input, state targets and using an observer for augmented states. Other characteristics of this algorithm include controllable expanded system model and handling model mismatch. By consideration of the state and input constraints, design of the MPC controller not only achieves the aim of offset-free control but also guarantees the constraint satisfaction in the presence of disturbance and model mismatch. The particular characteristics of the proposed algorithm are illustrated via a simulation using a continuous nonlinear jacketed stirred tank reactor (CSTR) model.

Key words: process control; offset-free control; model predictive control; constrained system; disturbance rejection

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0 Introduction

Model predictive control (MPC) is one of the important advanced control algorithms in both theory and practice. With features of handling physical constraints and the multi-variable problems, MPC has been widely applied in the industrial sector since 1978[1], meanwhile, MPC was expanded to theoretical field to improve and advance its theoretical bases. We recommend for details about various MPC algorithms[2-5].

An important task for achieving the good performance of the system, or producing high quality productions, is offset-free tracking of the desired output[6]. The offset-free control of constrained systems, in the presence of asymptotically constant disturbance and plant-model mismatch, is a very important subject therein [7-12]. Usually, to fulfill this goal, the system state space model is augmented by dynamically estimating the disturbance[13-14], and the target calculation is needed to compute the steady-state targets of the system input and state to track the piecewise constant setpoints[7-8]. The calculated target may vary by the estimated disturbance to remove its influence on the controlled variables. At each sampling time, by incorporating the calculated steady-state target into the optimization problem, MPC algorithm obtains the optimal control input sequence from which the first element is implemented, which steers the controlled variables to track the corresponding setpoints[14]. Conditions for closed-loop stability and constraint satisfaction have been presented in Refs. [2-5]. Also, a brief review of various types of MPC control laws in robust MPC is presented in Ref. [15] from which type 2 is applied to the algorithm proposed in this paper. Using reference governor strategy and predictive reference management, the tracking of constant reference in the absence of disturbance for constrained linear and nonlinear systems are addressed in Refs. [16-17], respectively. The algorithms presented in Refs. [6-8,13,18-21] for nonlinear and linear systems, respectively, using the linear models to guarantee the offset-free tracking of piecewise constant reference input in the presence of asymptotically constant disturbance. However, these algorithms have a high cost of computation, because the input and state targets need to be calculated according to the desired reference input and estimated disturbance at each sampling time. Additionally, determination of the steady-state and input targets based on the estimated disturbance, corresponding to model uncertainty and disturbance entering the plant, may violate the constraints or fail[8]. It has been emphasized that the offset-free control algorithms, which utilize different disturbance models and observer gains for the expanded system, leads to different closed loop performances[20-21]. This means that using different disturbance models along the different observers may result in an undesirable closed loop performance. In Refs. [7,18], there are some free parameters in the disturbance model that must be selected by designer and different selections of these parameters influence the performance and characteristics such as observability.

This paper addresses the offset-free MPC algorithm for constrained nonlinear systems, which is achievable for the piecewise constant steady-state target and immeasurable but bounded constant disturbance. The algorithm presents a new expanded system structure and predictive model in which the reference input is considered. Specifically, it is divided into two parts. The first part involves designing the linear time-invariant controller in order to stabilize the expanded system. Offset-free tracking performance is achieved by selecting a predictive model and designing the dynamic predictive controller. Optimization problem in the second part is solved over a stable closed loop system with consideration of state and input constraints. By using this algorithm in this paper, estimating the disturbances and calculating the input and state targets are unnecessary. There is no need to use an observer for augmented states because the new augmented states in each step can be computed, directly, using the measured output. In addition, the expanded system forms uniquely. Also, controllability of the expanded system, which is a necessary condition of the proposed algorithm, is guaranteed.

The remainder of this paper is organized as follows. By defining the problem in Section 2, Section 3 follows with introducing a strategy for offset-free tracking control. Then it will be shown how a model predictive control can be used to improve the linear controller and to satisfy the constraints. In Section 4, the main characteristics of the proposed predictive controller are addressed. Section 5 illustrates the performance of the proposed algorithm by an example of a nonlinear continuous stirred-tank reactor model.

The used notations are shown in Table 1.

Table 1　Notations

Symbol	Meaning
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{m\times n}$	$m\times n$-dimensional real matrix set
$\sim$	Pontryagin difference, i. e. , $\mathbb{A}\sim\mathbb{B}=\{a\mid a+b\in\mathbb{A},\forall b\in\mathbb{B}\}$
$\oplus_\square$	Minkowski sum, i. e. , $\mathbb{A}\oplus_\square\mathbb{B}=\{a+b\mid\forall a\in\mathbb{A},b\in\mathbb{B}\}$
$\oplus_{i\in\mathbb{N}_s}A_i$	$A_1\oplus A_2\oplus\cdots\oplus A_{\mathcal{N}_s}$
$x(t\mid k)$	the value of x at the future time $k+t$, predicted at the t th instant
x^{ss}	the steady state value of variable x
$[x;y]$	$[x^{\mathrm{T}},y^{\mathrm{T}}]^{\mathrm{T}}$
$v^*(k)$	optimal solution of optimization problem

1　Problem statement

Consider the discrete-time nonlinear system

$$\begin{cases}x_p(k+1)=f(x_p(k),u(k),w_p(k)),\\ y_p(k)=Cx_p(k),\end{cases}\tag{1}$$

with the constraints $Ex_p(k)+Fu(k)\leqslant\psi$, where $x_p\in\mathbb{R}^n$ is the measured state vector, $y_p\in\mathbb{R}^p$ is the controlled variable vector, $u\in\mathbb{R}^m$ is the control input, and $w_p\in\mathbb{R}^n$ is an immeasurable, asymptotically constant disturbance. $E\in\mathbb{R}^{s\times n}$, $F\in\mathbb{R}^{s\times m}$, $\psi\in\mathbb{R}^s$, and s is the number of linear constraints.

For the controller design, the following linear model is employed:

$$\begin{cases}x(k+1)=Ax(k)+Bu(k)+Dw(k),\\ y(k)=Cx(k),\end{cases}\tag{2}$$

where $x\in\mathbb{R}^n$, $y\in\mathbb{R}^p$, $u\in\mathbb{R}^m$. At each sampling time k, if $x_p(k)=x(k)$, it is apparent that $Dw(k)=f(x_p(k),u(k),w_p(k))-Ax(k)-Bu(k)$, where A is the state matrix, B the input matrix, C the output matrix, and D the immeasurable disturbance matrix. In this paper, we assume that $w(k)$ belongs to a bounded set $\mathcal{W}$ containing the origin, i. e. ,

$$w\in\mathcal{W}=\{w\in\mathbb{R}^q,\ \|w\|_\infty\leqslant\eta\}.\tag{3}$$

The goal of this paper is the control of system (1) so that the output tracks, offset-freely, setpoints in the presence of disturbances while the state and input satisfy the following constraints:

$$z=Ex+Fu\in Z:=\{z\in\mathbb{R}^s\mid z\leqslant\psi\}.\tag{4}$$

Assumption 1　That set Z is nonempty convex and compact, and contains the state and input corresponding to the steady-state setpoint in its interior.

Assumption 2　That state x_p is measurable, (A,B) is controllable, and

$$rank\begin{bmatrix}I-A & -B\\ C & 0\end{bmatrix}=n+p.\tag{5}$$

Remark 1　Equation (5) shows a necessary condition for (2) that guarantees controlled variables y to offset-freely track a setpoint. It is clear that, according to (5), the number of controlled variables cannot exceed the number of control inputs and the number of state variables, which means $p\leqslant\min\{m,n\}$[6,20]. In addition, (5) also implies that C has full row rank, namely, the controlled variables are independent of each other.

To guarantee the offset-free tracking, define

$$\begin{aligned}x_a(k+1)&=-Cx(k)+x_a(k)+y_s(k)\\&=-y(k)+x_a(k)+y_s(k).\end{aligned}\tag{6}$$

The expanded system is introduced

$$\begin{cases}\tilde{x}(k+1)=\tilde{A}\tilde{x}(k)+\tilde{B}u(k)+\tilde{d}(k),\\ y(k)=\tilde{C}\tilde{x}(k).\end{cases}\tag{7}$$

$$\tilde{A}=\begin{bmatrix}A & 0\\ -C & I\end{bmatrix},\ \tilde{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\ \tilde{C}=[C\quad 0],$$

$$\tilde{x}(k)=\begin{bmatrix}(x(k)\\x_a(k)\end{bmatrix},\ \tilde{d}(k)=\begin{bmatrix}0\\I\end{bmatrix}y_s(k),\qquad(8)$$

where $x_a\in\mathbb{R}^p$ is the augmented state vector and p is the number of controlled variables and $y_s\in\mathbb{R}^p$ is steady-state setpoint vector belonging to a bounded set Y_s.

Remark 2 In practice, the model mismatch between system and model or unknown disturbance inevitably exists, which may lead to an undesired offset for the controlled output at steady state. There exist two standard approaches to eliminate the offset: ① augmenting the process model to include a constant step disturbance; ② introducing integrate actions to design MPC controller[18,22]. One advantage of introducing integrate actions is to avoid designing an observer for estimating disturbance states, which simplifies the design for the proposed offset-free MPC method.

Lemma 1 (Controllability of expanded system) The expanded system (7) is controllable if and only if the controllability of (A,B) and rank condition (5) in Assumption 2 holds.

Proof According to the Hautus controllability condition, the expanded system (7) is controllable if

$$rank\begin{bmatrix}A-\lambda I & 0 & B\\-C & I-\lambda I & 0\end{bmatrix}=n+p:\forall\lambda.\qquad(9)$$

Using again the Hautus controllability condition, (A,B) is controllable if the first set of rows in (9) is linearly independent. For $\forall\lambda\neq1$, the second set of rows is linearly independent from each other and from first n rows. For $\lambda=1$, (9) remains full row rank if the rank condition (5) holds.

2 Offset free control strategy

In this section, as shown in Fig. 1, we mainly aim to design the offset-free MPC control strategy for constrained nonlinear system (1) in the presence of model mismatch and disturbance. The robust MPC controller design that explicitly considers the cost function, augmented model and physical constraints is presented in the following subsections.

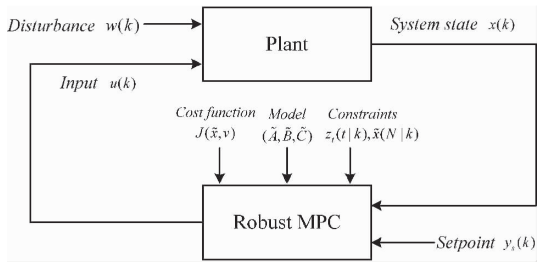

Fig. 1 Offset-free MPC strategy

2.1 Design of a stabilizing linear time-invariant controller

Suppose $u(k)=K\tilde{x}(k)$. Let $y_s\equiv0$ in (7). Consider performance index

$$J=\sum_{k=0}^{\infty}[\tilde{x}(k)^{T}Q\tilde{x}(k)+u(k)^{T}Ru(k)],\qquad(10)$$

where Q and R are symmetric nonnegative and symmetric positive-definite matrices, respectively.

By solving the unconstrained LQR problem for the expanded system (7), K will be obtained by

$$K=(R+\tilde{B}^{T}P\tilde{B})^{-1}\tilde{B}^{T}P\tilde{A},\qquad(11)$$

where P satisfies the Riccati equation:

$$P=Q+\tilde{A}^{T}P\tilde{A}-\tilde{A}^{T}P\tilde{B}(R+\tilde{B}TP\tilde{B})^{-1}\tilde{B}^{T}P\tilde{A}.\qquad(12)$$

The matrix K can be partitioned into two sub-matrices K_1 and K_2 with n and p columns of the matrix K, respectively,

$$K=[K_1\quad K_2)].\qquad(13)$$

Remark 3 In this paper, feedback gain matrix K is determined via the LQR problem. Besides, K can also be determined using any robust control strategy such as H_∞ in order to add more robustness to the control system.

2.2 Design of the dynamic predictive controller

The control move is composed of the state feedback and the perturbation item, i. e. ,

$$u(k)=K\tilde{x}(k)+v(k).\qquad(14)$$

By forming a predictive model and an optimization problem, the optimal value of $v(k)$ is calculated via a finite horizon optimization problem using model predictive control strategy.

2.2.1 The structure of the predictive model

The prediction model must be able to predict the future states of a system based on the input and current state of the system over the finite prediction horizon. In this paper, the prediction model is based on the expanded system (7), i. e.,

$$\tilde{x}(t+1|k)=\phi_d\tilde{x}(t|k)+\tilde{B}v(t|k)+\tilde{d}(k+t),$$
$$t=0,1,2,\cdots,N-1,$$
$$\tilde{x}(t|k)=\begin{bmatrix}\breve{x}(t|k)\\ x_a(t|k)\end{bmatrix},$$
$$\breve{x}(0|k)=x(k), \tag{15}$$

where $\tilde{x}(t|k)$ denotes the disturbance-free predicted expanded state vector along the prediction horizon, $\phi_d=\tilde{A}+\tilde{B}K$. Accordingly, $\tilde{u}(t|k)=K\tilde{x}(t|k)+v(t|k)$. The integer N is called the prediction horizon, which represents the number of free control perturbation moves $v(t|k)$ considered in the optimization problem.

2.2.2　Structure of constraints

The state and input constraints (4) should be satisfied for the generally nonlinear system (1) in the presence of disturbance w_p. The constraint is guaranteed in an optimization problem, which is based on the linear predictive model (15) corresponding to an expanded linear system (7), in the dynamic predictive control strategy. Therefore, satisfaction of the constraint (4) with respect to nonlinear system (1) is equivalent to guaranteeing the satisfaction of constraint (4) with respect to linear system (2) in the presence of any disturbance and model uncertainty, i. e., for all $w(\cdot)\in W$.

Lemma 2　Suppose $\tilde{x}(t|k)$ is the predicted state at the time step k using the prediction model (15).

Let $z_t(t|k)=\tilde{E}\tilde{x}(t|k)+Fu(t|k)\in Z_t:=\{z_t(t|k)\in\mathbb{R}^s|z_t(t|k)\leqslant\psi'\}$, $t=1,2,\cdots N-1$, where $\tilde{E}=[E\quad 0_{s\times p}]$, $\psi'\in\mathbb{R}^s$. The constraint (4) will be satisfied respect to system (1), if $z_t(t|k)$ belongs to the admissible set:

$$Z_t=Z\sim\bigoplus_{i=1}^{t}(E+FK_1)\phi_1^{i-1}DW, \tag{16}$$

where $\phi_1=A+BK_1$.

Proof　See A.

2.2.3　Terminal constraint and terminal cost

To guarantee constraint satisfaction and stability of the model predictive control, a suitable terminal constraint and a terminal cost should be designed based on the stabilizing linear controller[23]. In addition to the constraints arising by the physical and operational conditions, the following is also applied as a terminal constraint:

$$\chi_f:=\{\tilde{x}(t|k)\in\mathbb{R}^{n_d}|Y\tilde{x}(t|k)<\varphi,t\geqslant N\}, \tag{17}$$

where $Y\in\mathbb{R}^{r\times n_d}$, and $\varphi\in\mathbb{R}^r$ define the set χ_f.

According to Refs. [14,23－25], the terminal constraint χ_f is chosen such that

(1) χ_f is a subset of state space for which the constraint (4) is satisfied under the control $u=K\tilde{x}$, i. e.,

$$\chi_f\subset X:=\{\tilde{x}\in\mathbb{R}^{n_d}|(x,K\tilde{x})\in Z\}. \tag{18}$$

(2) χ_f is a robust invariant set for the closed-loop disturbed system $\tilde{x}(k+1)=\phi_d\tilde{x}(k)+D_dw(k)$, i. e.,

$$\tilde{x}(k+1)=\phi_d\tilde{x}(k)+D_dw(k)\in\chi_f,$$
$$\forall\tilde{x}\in\chi_f,\ \forall w\in W, \tag{19}$$

where $D_d=[D^{\mathrm{T}}\quad 0_{q\times p}^{\mathrm{T}}]^{\mathrm{T}}$.

A method of computing an χ_f has been reported by Ref. [25].

Assumption 3　(Terminal cost) Consider the feedback gain K and positive-definite matrix P, satisfying (11) and (12). Then according to Refs. [25], $V(\tilde{x}):=\tilde{x}^{\mathrm{T}}P\tilde{x}$ can be chosen as terminal cost. Following [26], $V(\tilde{x})$ is a Lyapunov function in χ_f for the undisturbed closed-loop system $\tilde{x}(k+1)=\phi_d\tilde{x}(k)$, i. e.,

$$V(\phi_d\tilde{x})-V(\tilde{x})\leqslant-\tilde{x}^{\mathrm{T}}(Q+K^{\mathrm{T}}RK)\tilde{x},\ \forall\tilde{x}\in\chi_f. \tag{20}$$

2.2.4　Quadratic program and dynamic predictive control

By incorporating the predictive dynamics, the state and input constraints, and the terminal constraint, the corresponding MPC algorithm is formulated as

$$\min_{v(t|k)}J(\tilde{x},v)=\sum_{t=0}^{N-1}(\|\tilde{x}(t|k)\|_Q^2+\|v(t|k)\|_R^2)+\|\tilde{x}(N|k)\|_P^2, \tag{21}$$

subject to

$$\begin{cases}(15),u(t|k)=K\tilde{x}(t|k)+v(t|k),\\ z_t(t|k)\in Z_t,\\ \tilde{x}(N|k)\in\chi_f,\end{cases} \tag{22}$$

where $t=0,1,\cdots,N-1$, P is the weighting matrix of terminal cost satisfying (12).

Remark 4 From another point of view referring to Lemma 1, the augmented state x_a is controllable and can be controlled for adding some new performance properties to the closed-loop control system. For example, the integrating modes in control systems may cause windup in the closed-loop systems[18], which can be damped by increasing the weighting parameters corresponding to the augmented state x_a (the integrator state) in the weighting matrix Q of index function (21) when the input is closed to its constraint boundary. Moreover, it is notable that filtering the steady-state setpoint y_s considered in vector $\tilde{d}$ in (15) is another way to decrease the influence of the windup.

Let

$$v^*(k) \triangleq \{v^*(0|k), v^*(1|k), \cdots, v^*(N-1|k)\} \tag{23}$$

be the optimal solutions of the optimization problem composed of (21) and (22). The control law is formed as below:

$$u(k) = K\tilde{x}(k) + v^*(0|k) \tag{24}$$

and is applied to the system (1).

Based on the above the discussions, the implementation of the proposed offset-free MPC method can be summarized in the following algorithm:

Algorithm 1 Offset-free MPC algorithm

(1) Initially, choose $\tilde{x}(0)$ and $y_s(0)$.

(2) Calculate the feedback gain matrix K by solving the unconstrained LQR problem (10).

(3) Calculate the optimal free control perturbation sequence $v^*(k)$ by solving the constrained optimization problem (21) ~ (22).

(4) Compute the control law $u(k)$ according to (24) and implement it to the system (1).

(5) Let $k=k+1$ and measure the $y_s(k)$, Then repeat the steps (3) ~ (4) until the algorithm ends.

The objective function for optimization problem (21) ~ (22) are quadratic functions with respect to the optimization variables $\{v^*(0|k), v^*(1|k), \cdots, v^*(N-1|k)\}$. However, the common quadratic programming method cannot be applied since the constraints $z_t(t|k) \in Z_t, t=1,2,\cdots,N-1$ and $\tilde{x}(N|k) \in \chi_f$ in (22) involve computational geometry method. Hence, the multi-parametric toolbox (MPT), which is designed for parametric optimization, geometry computation and MPC, is selected to solve the Algorithm in Section 2.2.4.

3 Properties of the predictive control

In this section, the following two features for offset-free MPC are detailed:

(1) Recursive feasibility: the optimization problem is always feasible with constraint satisfactory; and

(2) Closed-loop stability: the steady-state output of the system asymptotically converges to the setpoint in the presence of bound constant disturbance.

For an MPC optimization problem, feasibility is a fundamental property. Optimization with rather large control horizon over open loop input sequences may be infeasible especially in the presence of the disturbance[6]. However, there is not the same deficiency when optimization problem is solved with respect to the perturbation, which is added to the stabilizing feedback control law. Thus, according to Refs. [6,26], it is indicated that the finite horizon optimization problem (21) ~ (22) is feasible for all future time steps, if it is feasible at the first sample time $k=0$.

For the closed-loop system

$$\begin{cases} \tilde{x}(k+1) = \phi_d \tilde{x}(k) + \tilde{d}(k), \\ y(k) = \tilde{C}\tilde{x}(k), \end{cases} \tag{25}$$

the maximal constraint-admissible robust positively invariant set $\mathcal{O}_\infty$ is defined as

$$\mathcal{O}_\infty := \{\tilde{x}(0) \in \mathcal{X} | \tilde{x}(k+1) \in \mathcal{X}, \forall w \in \mathcal{W}, \forall k \geqslant 0\}, \tag{26}$$

where $\mathcal{X} | \tilde{x}(k+1) = \phi_d \tilde{x}(k) + \tilde{d}(k) + D_d w(k)$.

Assumption 4 The set $\mathcal{O}_\infty$ is nonempty, and can be determined by a definite number of affine inequality constraints.

For $\mathcal{X}$ defined in (18), $\mathcal{O}_\infty$ (or an inner approximation to it) can be readily calculated for linear time-invariant system $\tilde{x}(k+1)=\phi_d\tilde{x}(k)+\tilde{d}(k)+D_d w(k)$[25].

Suppose the set $X_t^v, t\in\{1,\cdots,N\}$ as

$$X_t^v := \{z\in Z \mid \exists x_a \in \mathbb{R}^{n_d-n} \text{ such that } \mathcal{V}_t(\tilde{x})\neq\varnothing\}, \tag{27}$$

where $\mathcal{V}_N(\tilde{x})=\mathcal{V}_t(\tilde{x}), t=N$ is the admissible set of perturbation satisfying the constraints in the presence of assumed disturbance. The set $X_t^v, t=N$, contains those states for which one can initialize the augmented state x_a in such a manner that the set of admissible input perturbations $V_N(\tilde{x})$ is nonempty. Thus the finite optimization problem (22) would be feasible.

Theorem 1 (Domain of predictive control law) Suppose that Assumptions 2 ~ 4 hold. If X_0 is defined as

$$X_0 := \{x\in\mathbb{R}^n \mid \exists x_a\in\mathbb{R}^p \text{ such that } \tilde{x}\in\mathcal{O}_\infty\} \tag{28}$$

and each X_t^v is defined as (27), then each set in $\{X_0, X_1^v,\cdots,X_N^v\}$ contains the state target in its interior and satisfies

$$X_0\subseteq X_1^v\cdots\subseteq X_{N-1}^v\subseteq X_N^v. \tag{29}$$

Proof See B.

Remark 5 Theorem 1 indicates that, under the mentioned assumptions, not only is set X_N^v nonempty, but also the size of the set of the initial states for which the offset-free control is achievable does not decrease with the increment of the horizon length.

Theorem 2 Suppose that Assumptions 2 ~ 4 hold, and the Algorithm 1 in Section 2.2.4 is applied. The closed-loop system $x(k+1)=Ax(k)+BK_N(\tilde{x}(k))+Dw(k)$ or the equivalent nonlinear system $x_p(k+1)=f(x_p(k),K_N(\tilde{x}(k)),w_p(k))$ is input to state stable (ISS) in X_N^v and the constraints (4) are satisfied for all time and for all disturbance sequences $w(\cdot)\in W$.

Proof See C.

Theorem 3 If MPC problem (21) ~ (22) is feasible and the closed-loop system to which the control input (14) is applied is stable, then the steady state output of the system (1) will converge to the piecewise constant setpoint, y_s, asymptotically.

Proof See D.

4 Simulation study

We consider a benchmark example, i. e., a CSTR in which an irreversible exothermic reaction $A\to B$ occurs, which is described by the following dynamic model based on a component balance for reactant A and an energy balance[27-28] (see Fig. 2):

$$\dot{C}_A=\frac{q}{V}(C_{Af}-C_A)-k_0\exp\left(-\frac{E}{RT}\right)C_A,$$

$$\dot{T}=\frac{q}{V}(T_f-T)+\frac{(-\Delta H)}{\rho C_p}k_0\exp\left(-\frac{E}{RT}\right)C_A+\frac{UA}{V\rho C_p}(T_c-T), \tag{30}$$

where C_A is the concentration of A in the reactor, T is the measurable reactor temperature, T_c is the temperature of the coolant stream. T_f is the temperature of the feed stream for which the deviation of nominal value is considered as process disturbance. The objective is to regulate C_A and T by manipulating T_c.

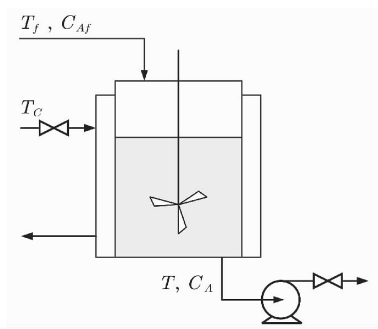

Fig. 2 CSTR

The nominal operating conditions correspond to the equilibrium $C_A^{eq}=0.5$ mol/L, $T^{eq}=350$ K, $T_c^{eq}=300$ K. Other parameters are listed in Table 2. Applying the sampling time $T_s=0.1$ min, the discrete time linear model to be applied in control algorithm is formed as

$$\begin{bmatrix}C_A(k+1)\\T(k+1)\end{bmatrix}=\begin{bmatrix}0.8 & -0.0035\\20.9192 & 1.4379\end{bmatrix}\begin{bmatrix}C_A(k)\\T(k)\end{bmatrix}+\begin{bmatrix}0\\0.2092\end{bmatrix}T_c(k),$$

$$y(k) = [0 \quad 1]\begin{bmatrix} C_A(k) \\ T(k) \end{bmatrix}. \qquad (31)$$

Table 2 Parameters for nonlinear model (30)

Parameter	Explanation	Value	Unit
T_f	feed temperature	350	K
q	flow rate	100	L/sec
C_{Af}	feed concentration	1	mol/L
V	reactor volume	100	L
k_0	reaction rate constant	7.2×10^{10}	$\min^{-1}$
UA	heat transfer coefficient	50 000	J/(sec K)
ρ	liquid density	1 000	g/L
C_p	heat capacity	0.239	J/(g K)
$(-\Delta H)$	reaction heat	50 000	J/mol
E/R	activation energy	8 750	K

The state matrix in the above unstable linear model has one stable and one unstable eigenvalues. The controlled variable of the nonlinear system is the reactor temperature T, which should track the setpoint T_r without offset in the presence of disturbance. During the offset-free tracking control, the following temperature constraint must be satisfied to maintain the feasible operation: 280 K $\leqslant T_c \leqslant$ 370 K, 280 K $\leqslant T \leqslant$ 370 K, $0 \leqslant C_A \leqslant 1$ mol/L.

The proposed algorithm applies the same fixed prediction and control horizon, $N = 8$, the state penalty matrices, $Q = 10^3\, I$ for linear control design and predictive control design, and control input penalty $R = 1$ have been applied.

At first, the proposed algorithm initiates the offset-free tracking feature using novel augmented states and stabilizing state feedback controller. The offset-free tracking task is completed by constructing a new predictive control and designing a dynamic predictive controller. In the controller design, the state and input constraints are considered and handled in the presence of disturbance and model mismatch, which means that the resulting controller is of constraint satisfactory.

The closed-loop simulation results of controlled variable $T(k)$ and input $T_c(k)$ with MPC controller, subject to piecewise constant setpoint, is presented in Fig. 3. The first diagram of Fig. 3 depicts the offset-free tracking of piecewise constant setpoint, the second shows the control input satisfying constraint, and the third illustrates the nominal value of feed temperature without disturbance.

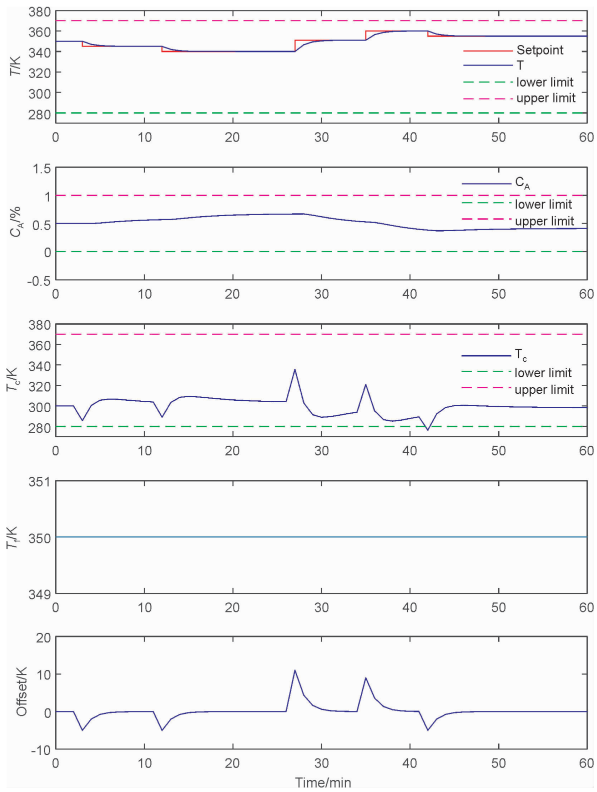

Fig. 3 Closed-loop offset-free proposed controller subject to piecewise constant setpoint

In Fig. 4, we present the closed-loop offset-free proposed controller rejecting the piecewise constant disturbance. The first diagram shows the offset-free control subject to constant setpoint rejecting the piecewise constant disturbance, the second depicts the control input satisfying constraint, and the varying piecewise constant constraint value of feed temperature considered as system disturbance is illustrated in the third diagram.

Fig. 5 shows the closed-loop offset-free proposed controller subject to piecewise constant setpoint in the presence of the disturbance. As can be seen from the Fig. 6, the offset-free tracking of piecewise constant setpoint rejecting the piecewise constant disturbance is in the first diagram, the second depicts the control input satisfying constraint, and the third illustrates the varying piecewise constant constraint value of feed temperature considered as system disturbance.

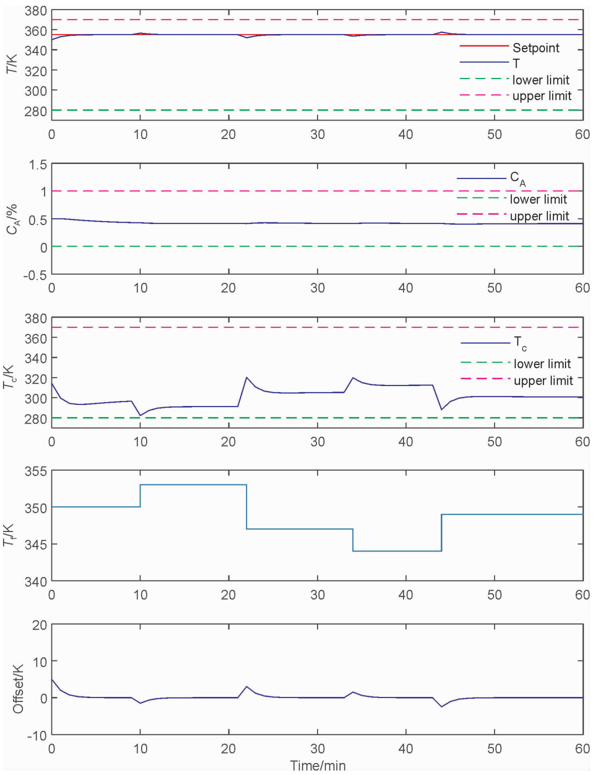

Fig. 4　Closed-loop offset-free proposed controller rejecting the piecewise constant disturbance

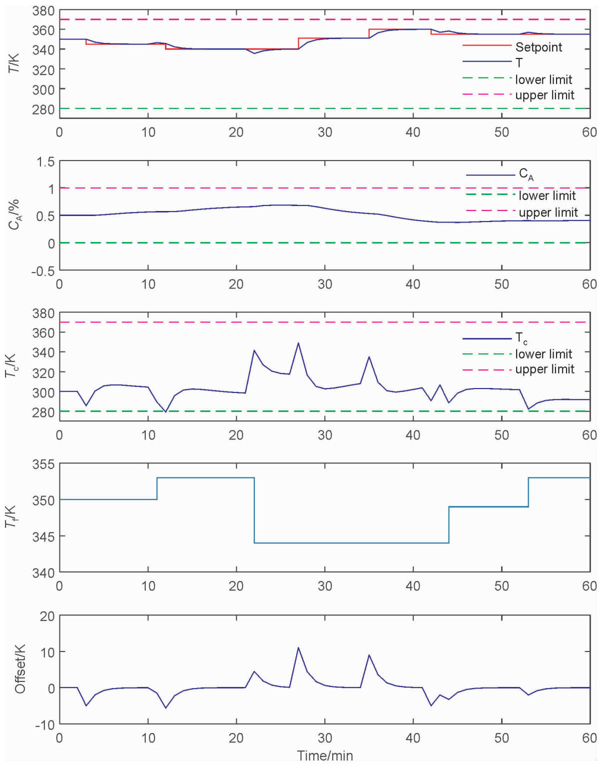

Fig. 5　Closed-loop offset-free proposed controller subject to piecewise constant setpoint in the presence of the disturbance

We finally present in Fig. 6 a non-offset-free robust MPC controller. As an example, we choose the approach in Ref. [29], which is similar to the one proposed in this paper, in the sense that a pre-stabilizing gain matrix is used and the plant state prediction at the end of the horizon is restricted in the maximal disturbance invariant set $\mathcal{O}_\infty$. Both controllers are based on the same stabilizing gain matrix K, which is the optimal LQR gain with the same Q and R. As seen from Fig. 6, the controller proposed in Ref. [29] does not guarantee offset-free control which leaves an undesired steady-state offset.

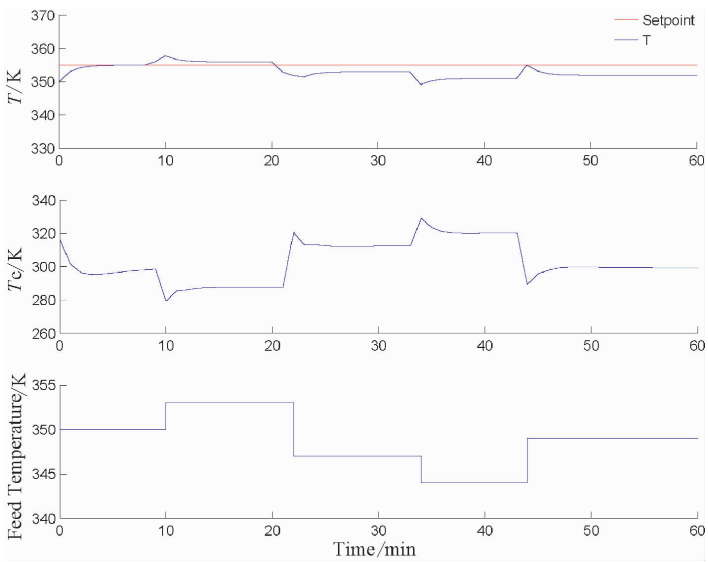

Fig. 6　Closed-loop of nonoffset-free controller

Despite most of the former algorithms, by using this algorithm there is no need for calculating the input and state targets and using an observer for augmented states because of the proposed novel structure for expanded system. Other characteristics of this algorithm include: the controllable expanded system without any free parameter to be chosen, handling model mismatch for nonlinear systems, no need for disturbance estimating and target calculating, feasible optimization, and less computation effort. Simulation results depict a reliable and desirable performance of the proposed algorithm for offset-free tracking of different constant set-points in the presence of piecewise constant disturbances.

5 Conclusions

For the nonlinear system subject to state and input constraints, the offset-free MPC problem is addressed in this paper. In the proposed algorithm, the artificial disturbance is introduced to replace the modeling error and construct a new expanded system. In order to achieve the offset-free control, the design of dual part control law consists of the linear stabilizing control and the dynamic predictive control. Compared with the previous techniques, the novelty of this paper is to achieve offset-free control without calculating the steady-state input, state targets, or designing an observer for augmented states. A special feature of this method is that the optimization problem becomes more feasible via optimizing the stabilized closed-loop system. In addition, by consideration of the state and input constraints, designing of the MPC controller not only achieves the aim of offset-free control but also guarantees the constraint satisfaction in the presence of disturbance and model mismatch.

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Appendix

A　Proof for Lemma 2

According to the equivalent linear model (2) and considering the control law (14) with the partitioned form (13), the prediction of x is

$$x(t+1|k)=\phi_1 x(t|k)+Bv(t|k)+BK_2x_a(t|k)+Dw(k+t),t=1,2,\cdots,N-1, \quad (A1)$$

where $\phi_1=A+BK_1$. (A1) is iterated over prediction horizon as

$$\begin{aligned}&x(0\mid k)=x(k),\ t=0,\\ &x(t\mid k)=\phi_1^t x(k)+\sum\nolimits_{i=1}^{t}\phi_1^{i-1}(Bv(t-i\mid k)+\\ &BK_2x_a(t-i\mid k)+Dw(k+t-i)),\ t=1,2,\cdots,N-1.\end{aligned} \quad (A2)$$

On the another hand, $\breve{x}(t|k)$ as a predicted state based on the prediction model (15), with respect to time step k, has the following form:

$$\breve{x}(t+1|k)=\phi_1\breve{x}(t|k)+Bv(t|k)+BK_2x_a(t|k),t=1,2,\cdots,N-1, \quad (A3)$$

which is iterated over prediction horizon as

$$\begin{aligned}&\breve{x}(0|k)=x(k),t=0,\\ &\breve{x}(t|k)=\phi_1^t x(k)+\sum_{i=1}^{t}\phi_1^{i-1}(Bv(t-i|k)+BK_2x_a(t-i|k)),t=1,2,\cdots,N-1.\end{aligned} \quad (A4)$$

Subtracting (A2) from (A4) yields

$$x(t\mid k)=\breve{x}(t\mid k)+\sum_{i=1}^{t}\phi_1^{i-1}Dw(k+t-i). \tag{A5}$$

For $u(t|k)$, substituting (A5) into $u(t|k)=K_1x(t|k)+K_2x_a(t|k)+v(t|k)$ obtains

$$u(t\mid k)=K\tilde{x}(t\mid k)+v(t\mid k)+K_1\sum_{i=1}^{t}\phi_1^{i-1}Dw(k+t-i)=$$
$$\tilde{u}(t\mid k)+K_1\sum_{i=1}^{t}\phi_1^{i-1}Dw(k+t-i). \tag{A6}$$

Followed from (A5) and (A6), $z(t\mid k)=z_t(t\mid k)+(E+FK_1)\sum_{i=1}^{t}\phi_1^{i-1}Dw(k+t-i)$ holds, which can be rewritten as

$$Z_t=Z\sim\bigoplus_{i=1}^{t}(E+FK_1)\phi_1^{i-1}DW. \tag{A7}$$

Therefore, the satisfaction of constraint (4) will be guaranteed if $z(t|k)\in Z_t$ associated with predictive model (15). Note that Z_t is non-empty set due to Assumption 1. The conclusion holds.

B Proof for Theorem 1

Suppose that Assumption 4 holds. Thus for $t=0$, the state target corresponding to the setpoint is included in the interior of X_0.

By induction, consider the state x where $x\in X_t^v$, $t\in\{1,\cdots,N-1\}$. Then the additional state x_a exists for which $V_t(\tilde{x})$ is nonempty and there is an admissible perturbation sequence $v_t=\{v_0,\cdots,v_(t-1)\}\in V_t(\tilde{x})$ that implies $\tilde{x}\in\mathcal{O}_\infty$ for all $w\in W$ defined by (3). According to (26), $\mathcal{O}$is disturbance invariant and constraint-admissible for the closed-loop system

$$\tilde{x}(k+1)=\phi_d\,\tilde{x}(k)+\tilde{d}(k). \tag{B8}$$

Hence, $\mathcal{O}_\infty$ is disturbance invariant and constraint-admissible for system

$$\tilde{x}(k+1)=\phi_d\,\tilde{x}(k)+\tilde{d}(k)+\tilde{B}v(k) \tag{B9}$$

under the infinite perturbation sequence $\{v(k),k=0,1,2,\cdots\}$. In other words, if $\tilde{x}(t)\in\mathcal{O}_\infty$ for all $w\in W$, then $\tilde{x}(t+1)\in\mathcal{O}_\infty$ for all $w\in W$. This implies that if $v_t\in V_t(\tilde{x})$, then $[v_t;0]\in V_{t+1}(\tilde{x})$. Hence, if $V_t(\tilde{x})$ is nonempty, then $V_{t+1}(\tilde{x})$ is nonempty. Thereby, according to the definition of X_t^v in (27), if $x\in X_t^v$, then $x\in X_{t+1}^v$, hence $X_t^v\subseteq X_{t+1}^v$.

Repeating the above argument, the proof is completed by noticing that $X_0\subseteq X_1^v$. The proof of this theorem is based on the theorem presented in Ref. [6].

C Proof for Theorem 2

For a system in the form of the expanded system (7), i. e., $\tilde{x}(k+1)=\tilde{A}\tilde{x}(k)+\tilde{B}K_N(\tilde{x}(k))+\begin{bmatrix}D & 0\\ 0 & I_p\end{bmatrix}\begin{bmatrix}w(k)\\ y_s(k)\end{bmatrix}$ and a feedback policy by the form of (14), all the results of this theorem have been reported and proved[26]. Since the state and control input of the system (2) is exactly the same as first n state and control input of the expanded system (according to the structure of the expanded system i. e. (7)), this theorem holds for the closed-loop system $x(k+1)=Ax(k)+BK_N(\tilde{x}(k))+Dw(k)$, $\forall w\in W$, and consequently, the results of this theorem holds for nonlinear closed loop system (1), i. e., $x_p(k+1)=f(x_p(k),K_N(\tilde{x}(k)),w_p(k))$.

D Proof for Theorem 3

The state of the stable closed-loop system corresponding to piecewise constant setpoint remains constant, i. e., $\tilde{x}^{ss}(k+1)=\tilde{x}^{ss}(k)=\tilde{x}^{ss},k\to\infty$. Therefore, at the steady-state, (6) can be rewritten as

$$x_a^{ss}=-Cx^{ss}+x_a^{ss}+y_s=-y^{ss}+x_a^{ss}+y_s \tag{D10}$$

and it implies that

$$y^{ss}=y_s,\ k\to\infty. \tag{D11}$$

In other words, the offset-free tracking of the piecewise constant setpoint, y_s, is guaranteed.