混合时滞惯性神经网络的固定时间投影同步

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摘　要　研究了一类带有混合时滞、状态切换连接权重、不连续激活函数的惯性神经网络的固定时间投影同步问题.利用非降阶法、不等式放缩技巧、微分包含理论和恰当的Lyapunov函数得到了此类系统固定时间投影同步判据.所构建的切换控制器有效解决了因系统跳变对同步研究带来的不确定性难题.最后,利用数值仿真验证本文结论的可靠性.

关键词　惯性神经网络;固定时间投影同步;混合时滞;非降阶方法;切换控制器
中图分类号　O625.67;O643.3　**文献标志码**　A　**文章编号**　1672-4321(2025)03-0407-07
doi:10.20056/j.cnki.ZNMDZK.20250315

Fixed-time projective synchronization of inertial neural network with mixed time delays

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Abstract　The problem of fixed-time projective synchronization of inertial neural networks with mixed time delays, state-switching connection weights and discontinuous activation functions(DAFs) is studied. By using non-reduction method, inequality reduction technique, differential inclusion theory and appropriate Lyapunov function, the fixed-time projective synchronization criterion of such systems is obtained. The constructed switching controller can effectively solve the uncertainty problem caused by system hopping. Finally, the reliability of the conclusions is verified by numerical simulation.

Keywords　inertial neural networks; fixed-time projective synchronization; mixed delays; non-reduction method; switching controller

近些年来,人工神经网络在模式识别[1]、联想记忆[2]、信息处理[3]及图像加密[4]等领域得到广泛应用,而这些应用的前提是对神经网络的结构和动力学行为的深入研究,比如稳定性[5]、无源性[6]、耗散性[7]和同步[8]等.

同时,同步作为神经网络重要的动力学行为一直受到广大学者的广泛关注和持续研究,相关成果被陆续报道出来,如文献[9-13].值得指出的是,上述这些研究只关注完全同步.事实上,驱动-响应系统按比例同步,即投影同步[14],它可以提供更高的网络安全性和更快的传输速度.近几年,投影同步逐渐开始受到研究者们的重视.另外,以往研究的大多数神经网络模型是由一阶微分方程来描述的[9,11],研究表明,一些生物神经元的轴突可以通过电感来模拟,这种加入了电感的神经网络模型称为惯性神经网络,其在图像处理、记忆的无序搜索等方面展现出巨大应用优势.然而,因惯性项的存在,使得惯性神经网络更易产生不稳定的振荡、混沌等复杂的动力学响应,研究此类系统的稳定、同步等动力学行为将为其在工程应用方面奠定基础.目前,关于惯性神经网络稳定、同步等的研究还有很多需要补充和完善的地方.本文针对带分布时滞的依状态切

收稿日期　2024-03-12
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基金项目　国家自然科学基金资助项目(62373383)

表1 本文中符号的解释

Tab. 1 Explanation of the symbols in this paper

符号	含义	符号	含义
N_+	正整数集	β	$\max_{k\in H}\{\beta_k\}$
$\mathcal{R}^s$	S维欧式空间	$C([t_0-\beta,t_0],R^d)$	$[t_0-\beta,t_0]\mapsto\mathcal{R}^d$的连续函数
H	$\{1,2,\cdots,n\}$	$C^1(\mathcal{R}^d,[0,+\infty))$	$\mathcal{R}^d\mapsto[0,+\infty)$上存在一阶偏导连续函数
1-norm	$\Vert p\Vert=\sum_{k=1}^{L}\lvert p_k\rvert$	$\max\varpi_k$	$\max\{\varpi_k^*,\varpi_k^{**}\}$
$co\{m,n\}$	由常数m和n组成的凸闭包	$\max\iota_{kr}$	$\max\{\lvert\iota_{kr}^*\rvert,\lvert\iota_{kr}^{**}\rvert\}$
$K[\Omega]$	集合Ω的凸闭包	$\max\varsigma_{kr}$	$\max\{\lvert\varsigma_{kr}^*\rvert,\lvert\varsigma_{kr}^{**}\rvert\}$
$\Gamma(j^-)$	点j的左极限	$\max o_{kr}$	$\max\{\lvert o_{kr}^*\rvert,\lvert o_{kr}^{**}\rvert\}$
$\Gamma(j^+)$	点j的右极限		

换惯性神经网络,对其固定时间同步问题进行深入研究,所得结果将进一步丰富和发展惯性神经网络的理论体系.

1 本文模型

1.1 模型建立

考虑如下一类带分布式时滞的惯性神经网络模型:

$$\frac{d^2(j_k(t))}{dt^2}=-\delta_k\frac{d(j_k(t))}{dt}-\varpi_k(j_k(t))j_k(t)+\sum_{r=1}^{L}\iota_{kr}(j_k(t))F_r(j_r(t))+\sum_{r=1}^{L}\varsigma_{kr}(j_k(t))F_r(j_r(t-\tau_r(t)))+\sum_{r=1}^{L}o_{kr}(j_k(t))\int_{t-m_r(t)}^{t}F_r(j_r(t))dt+I_k,k\in H,t\geqslant 0,\quad(1)$$

式中:$j_k(t)$代表k时刻的神经元状态;$\frac{d^2(j_k(t))}{dt^2}$是惯性项;$\delta_k>0$表示自反馈常数;$F_r(\cdot)$是不连续激活函数;$\tau_r(t)$和$m_r(t)$分别为离散时滞和分布时滞,其中$0\leqslant\tau_r(t)\leqslant\tau_r$,$0\leqslant m_r(t)\leqslant m_r$;$I_k$代表外部输入;$\varpi_k(j_k(t)),\iota_{kr}(j_k(t)),\varsigma_{kr}(j_k(t)),o_{kr}(j_k(t))$为状态切换连接权重,取值如下:

$$\varpi_k(j_k(t))=\begin{cases}\varpi_k^*,\lvert j_k(t)\rvert\leqslant\Gamma_k\\ \varpi_k^{**},\lvert j_k(t)\rvert>\Gamma_k\end{cases},\quad \iota_{kr}(j_k(t))=\begin{cases}\iota_k^*,\lvert j_k(t)\rvert\leqslant\Gamma_k\\ \iota_k^{**},\lvert j_k(t)\rvert>\Gamma_k\end{cases},\quad \varsigma_{kr}(j_k(t))=\begin{cases}\varsigma_{kr}^*,\lvert j_k(t)\rvert\leqslant\Gamma_k\\ \varsigma_{kr}^{**},\lvert j_k(t)\rvert>\Gamma_k\end{cases},\quad o_{kr}(j_k(t))=\begin{cases}o_{kr}^*,\lvert j_k(t)\rvert\leqslant\Gamma_k\\ o_{kr}^{**},\lvert j_k(t)\rvert>\Gamma_k\end{cases},\quad(2)$$

这里,切换开关$\Gamma_k>0,\varpi_k^*>0,\varpi_k^{**}>0$,并且$\iota_k^*$,$\iota_k^{**},\varsigma_{kr}^*,\varsigma_{kr}^{**},o_{kr}^*,o_{kr}^{**},k,r\in H$都是常数.惯性神经网络(1)的初始位置为$j_k(s)=\phi_k(s),\dot{j}_k(s)=\varphi_k(s)$,$k\in H,s\in[-\beta,0]$,且$\phi_k(s),\varphi_k(s)\in C([-\beta,0],\mathcal{R})$.

惯性神经网络(1)的响应系统如下:

$$\frac{d^2(l_k(t))}{dt^2}=-\delta_k\frac{d(l_k(t))}{dt}-\varpi_k(l_k(t))l_k(t)+\sum_{r=1}^{L}\iota_{kr}(l_k(t))F_r(l_r(t))+\sum_{r=1}^{L}\varsigma_{kr}(l_k(t))F_r(l_r(t-\tau_r(t)))+\sum_{r=1}^{L}o_{kr}(l_k(t))\int_{t-m_r(t)}^{t}F_r(l_r(t))dt+I_k+u_k(t),k\in H,t\geqslant 0,\quad(3)$$

这里,$l_k(t)$代表k时刻的神经元状态,$u_k(t)$代表控制器,其他系数与(1)式中相同.令惯性神经网络(3)的初始条件为:$l_k(s)=\chi_k(s),\dot{l}_k(s)=\psi_k(s),k\in H$,$s\in[-\beta,0]$,且$\chi_k(s),\psi_k(s)\in C([-\beta,0],\mathcal{R})$.

1.2 基本假设

假设1 对于$\forall r\in H,F_r(.)\in C(\mathcal{R}\backslash Q,\mathcal{R})$,其中集合$Q$由有限个数的不连续点$p_{q_r}^r$组成,其中$q_r\in N_+$,它具有右极限$F_r(p_{q_r}^{r+})$和左极限$F_r(p_{q_r}^{r-})$.

假设2 对于$\forall r\in H$,存在$M_r>0$,使得$F_r(.)\leqslant M_r$,并且有:

$\sup_{a_r\in K[F_r(\mp)],b_r\in K[F_r(\pm)]}\lvert a_r-b_r\rvert\leqslant\Lambda_r\lvert\mp-\pm\rvert+\Delta_r$,这里$\mp,\pm\in\mathcal{R},\Lambda_r\geqslant 0,\Delta_r\geqslant 0$,

$$K[F_r(\mp)]=\{\min\{F_r(\mp^-),F_r(\mp^+)\},\max\{F_r(\mp^-),F_r(\mp^+)\}\},$$

$$K[F_r(\pm)]=\{\min\{F_r(\pm^-),F_r(\pm^+)\},\max\{F_r(\pm^-),F_r(\pm^+)\}\}.$$

本文中,由于系统不连续,所以引入Filippov解.由微分包容理论,可得:

$$\frac{\mathrm{d}^2(j_k(t))}{\mathrm{d}t^2}+\delta_k\frac{\mathrm{d}\left(j_k(t)\right)}{\mathrm{d}t}\in -co\left[\varpi_k\left(j_k(t)\right)\right]j_k(t)+\sum_{r=1}^{L}co\left[\iota_{kr}(j_k(t))\right]K\left[F_r(j_r(t))\right]+\sum_{r=1}^{L}co\left[\varsigma_{kr}\left(j_k(t)\right)\right]K\left[F_r\left(j_r(t-\tau_r(t))\right)\right]+\sum_{r=1}^{L}co\left[o_{kr}\left(j_k(t)\right)\right]\int_{t-m_r(t)}^{t}K\left[F_r\left(j_r(t)\right)\right]\mathrm{d}t+I_k,\quad k\in H,t\geqslant 0,\tag{4}$$

即，存在 $\Phi_r(t)\in K\left[F_r(j_r(t))\right]$，$\varpi_k(t)\in co\left[\varpi_k(j_k(t)\right]=co\left[\varpi_k{}^{*},\varpi_k{}^{**}\right]$，$\iota_{kr}(t)\in co\left[\iota_{kr}(j_k(t)\right]=co\left[\iota_{kr}{}^{*},\iota_{kr}{}^{**}\right]$，$\varsigma_{kr}(t)\in co\left[\varsigma_{kr}(j_k(t)\right]=co\left[\varsigma_{kr}{}^{*},\varsigma_{kr}{}^{**}\right]$，$o_{kr}(t)\in co\left[o_{kr}(j_k(t)\right]=co\left[o_{kr}{}^{*},o_{kr}{}^{**}\right]$.

对于任意的 $t\geqslant 0$，有：

$$\frac{\mathrm{d}^2(j_k(t))}{\mathrm{d}t^2}+\delta_k\frac{\mathrm{d}\left(j_k(t)\right)}{\mathrm{d}t}\in -\varpi_k(t)j_k(t)+\sum_{r=1}^{L}\iota_{kr}(t)\Phi_r(t)+\sum_{r=1}^{L}\varsigma_{kr}(t)\Phi_r\left(t-\tau_r(t)\right)+\sum_{r=1}^{L}o_{kr}(t)\int_{t-m_r(t)}^{t}\Phi_r(t)\mathrm{d}t+I_k,\ k\in H,t\geqslant 0,\tag{5}$$

这里，当 $F_r(j_r(t)$ 在 $j_r(t)$ 处连续时，$K\left[F_r\left(j_r(t)\right)\right]=F_r\left(j_r(t)\right)$，$co\left[\varpi_k\left(j_k(t)\right)\right]=\varpi_k\left(j_k(t)\right)$.

定义1　函数 $\tilde{I}(t)=(\tilde{I}_1(t),\tilde{I}_2(t),\cdots,\tilde{I}_l(t))^{\mathrm{T}}$ 是惯性神经网络(1)的Filippov解，并且初始位置是 $J_k(s)=\phi_k(s),\dot{J}(s)=\varphi_k(s),k\in H,s\in\left[-\beta,0\right]$，且 $\phi_k(s),\varphi_k(s)\in C(\left[-\beta,0\right],\mathcal{R})$. 由于，$L$ 是紧区间，且 $\forall L\subset[0,+\infty)$，那么绝对连续函数 $\tilde{I}(t)$ 满足系统(4)和(5).

1.3　误差系统的表述和控制器的设计

令 $E_k(t)$ 表示误差系统，并且 $E_k(t)=l_k(t)-\varrho j_k(t)$，其中 ϱ 表示投影参数，$\varrho\in N_+,k\in H,t\geqslant 0$. 因此，得到误差系统的表达式：

$$\frac{\mathrm{d}^2E_k(t)}{\mathrm{d}t^2}=-\delta_k\frac{\mathrm{d}E_k(t)}{\mathrm{d}t^2}-\varpi_k\left(l_k(t)\right)l_k(t)+\varrho\varpi_k\left(j_k(t)\right)j_k(t)+\sum_{r=1}^{L}\left[\iota_{kr}\left(l_k\right)F_r\left(l_r(t)\right)-\varrho\iota_{kr}\left(j_k\right)F_r\left(j_r(t)\right)\right]+\sum_{r=1}^{L}\left[\varsigma_{kr}\left(l_k(t)\right)F_r\left(l_r\left(t-\tau_r(t)\right)\right)-\varrho\varsigma_{kr}\left(j_k(t)\right)F_r\left(j_r\left(t-\tau_r(t)\right)\right)\right]+\sum_{r=1}^{L}\left[o_{kr}\left(l_k(t)\right)\cdot\int_{t-m_r(t)}^{t}F_r\left(l_r(t)\right)\mathrm{d}t-\varrho o_{kr}\left(j_k(t)\right)\int_{t-m_r(t)}^{t}F_r\left(j_r(t)\right)\right]\mathrm{d}t+\left(1-\varrho\right)I_k(t)+u_k(t),\ t\geqslant 0,k\in H,\tag{6}$$

杂合控制器 $u_k(t)$ 设计如下：

$$u_k(t)=\nu_k(t)+\omega_k(t),\tag{7}$$

$\nu_k(t)$ 是带有状态切换的杂合控制器：

$$\nu_k(t)=\begin{cases}\sum_{r=1}^{L}\varrho\iota^{*}F_r\left(j_r(t)\right)-\sum_{r=1}^{L}\iota^{*}F_r\left(\varrho(j_r(t))\right), \text{若}\left|j_k(\mathrm{t})\right|\leqslant\Gamma_k,\left|l_k(t)\right|\leqslant\Gamma_k,\\ \sum_{r=1}^{L}\varrho\iota^{**}F_r\left(j_r(t)\right)-\sum_{r=1}^{L}\iota^{**}F_r\left(\varrho(j_r(t))\right), \text{若}\left|j_k(\mathrm{t})\right|>\Gamma_k,\left|l_k(t)\right|>\Gamma_k,\\ \sum_{r=1}^{L}\varrho\iota^{*}F_r\left(j_r(t)\right)-\sum_{r=1}^{L}\iota^{**}F_r\left(\varrho(j_r(t))\right)+\varrho(\varpi_k^{**}-\varpi_k^{*})j_k(t), \text{若}\left|j_k(\mathrm{t})\right|\leqslant\Gamma_k,\left|l_k(t)\right|>\Gamma_k,\\ \sum_{r=1}^{L}\varrho\iota^{**}F_r\left(j_r(t)\right)-\sum_{r=1}^{L}\iota^{*}F_r\left(\varrho(j_r(t))\right)+\varrho(\varpi_k^{*}-\varpi_k^{**})j_k(t), \text{若}\left|j_k(\mathrm{t})\right|>\Gamma_k,\left|l_k(t)\right|\leqslant\Gamma_k.\end{cases}\tag{8}$$

$$\omega_k(t)=-\xi_kE_k(t)-\zeta_k\dot{E}_k(t)-sign(\dot{E}_k(t))\left[\rho_k+\theta_k\left|E_k(t)\right|^{\varepsilon}+\nu_k\left|\dot{E}_k(t)\right|^{\varepsilon}+\gamma_k\left|E_k(t)\right|^{\tau}+\mu_k\left|\dot{E}_k(t)\right|^{\tau}\right],\tag{9}$$

这里 $0<\varepsilon<1,\tau>1$，同时 $\xi_k,\zeta_k,\theta_k,\nu_k,\gamma_k,\mu_k,\rho_k>0,k\in H$.

注1　控制器(7)去掉了文献[11]中的限制条件，同时，还可以简化成如下形式：

$$u_k(t)=\hat{\nu}_k(t)+\omega_k(t),\tag{10}$$

$$\hat{\nu}_k(t)=\begin{cases}\varrho\left(\varpi_k^{**}-\varpi_k^{*}\right)j_k(\mathrm{t}),\text{若}\left|j_k(\mathrm{t})\right|\leqslant\Gamma_k,\left|l_k(t)\right|>\Gamma_k,\\ \varrho\left(\varpi_k^{**}-\varpi_k^{*}\right)j_k(\mathrm{t}),\text{若}\left|j_k(\mathrm{t})\right|>\Gamma_k,\left|l_k(t)\right|\leqslant\Gamma_k,\end{cases}\tag{11}$$

其中，$\omega_k(t)$ 保持不变.

1.4　投影同步的定义和引理

定义2　如果 $T\left(l^{*}(0),\tilde{j}(0)\right)>0$ 成立，这里存在镇定时间 $T_{\max}>0$，使得 $T\left(l^{*}(0),\tilde{j}(0)\right)\leqslant T_{\max}$，并且 $\lim\limits_{t\to T_{\max}}\left\|l^{*}(t)-\varrho\tilde{j}(t)\right\|=0$，其中：

$$l^{*}(t)=\left(l_1(t),l_2(t),\cdots l_l(t),\ \dot{l}_1(t),\ \dot{l}_2(t),\cdots,\ \dot{l}_l(t)\right)^{\mathrm{T}},$$

$$\tilde{j}(t)=\left(j_1(t),j_2(t),\cdots j_l(t),\ \dot{j}_1(t),\ \dot{j}_2(t),\cdots,\ \dot{j}_l(t)\right)^{\mathrm{T}},$$

$$l_k(t),\ \dot{l}_k(t),j_k(t),\ \dot{j}_k(t)\in\mathcal{R},k\in H,t\geqslant 0,$$

那么驱动系统(1)和响应系统(3)就称为固定时间投影同步.

引理1[14]　下面的微分包含系统可以表示在杂合控制器 $u_k(t)$ 控制下的误差系统(6).

$$\frac{\mathrm{d}^2E_k(t)}{\mathrm{d}t^2}+\delta_k\frac{\mathrm{d}E_k(t)}{\mathrm{d}t^2}\in -co(\varpi_k^{*}-\varpi_k^{**})E_k(t)+\sum_{r=1}^{L}co(\iota_{kr}^{*},\iota_{kr}^{**})\times K\left[F_r\left(l_r(t)\right)-F_r\left(\varrho(j_r(t))\right)\right]+\sum_{r=1}^{L}co(\varsigma_{kr}^{*},\varsigma_{kr}^{**})\times K\left[F_r\left(l_r\left(t-\tau_r(t)\right)\right)-F_r\left(\varrho(j_r\left(t-\tau_r(t)\right))\right)\right]+$$

$$\sum_{r=1}^{L} co(o_{kr}^{*}, o_{kr}^{**}) \times \int_{t-m_r(t)}^{t} K\left[F_r\left(l_r(t)\right) - F_r\left(\varrho(j_r(t))\right)\right]\mathrm{d}t + K[\omega_k(t)], t \geqslant 0, k \in H. \quad (12)$$

引理2[15] 假设$V(.)$在$C^1(\mathcal{R}^{2d}, [0, +\infty))$上径向无界,且$V\left(\tilde{E}(t)\right) = 0 \rightleftharpoons \tilde{E}(t) = 0$,并且系统(6)的解满足$\frac{\mathrm{d}V(\tilde{E}(t))}{\mathrm{d}t} \leqslant -\epsilon + \upsilon V\left(\tilde{E}(t)\right) - \Im V\left(\tilde{E}(t)\right)^{\varepsilon} - hV\left(\tilde{E}(t)\right)^{\sigma}$,那么驱动系统(1)和响应系统(3)达到固定时间投影同步,其中$\Im, h, \upsilon, \epsilon > 0, 0 < \varepsilon < 1, \sigma > 1$,$\upsilon < \min\{\Im, h\}$,$\tilde{E}(t) = (E_1(t), E_2(t), \cdots, E_l(t), \dot{E}_1(t), \dot{E}_2(t), \cdots, \dot{E}_l(t))^{\mathrm{T}}$.镇定时间的计算公式如下:

$$T_{\max} = \frac{2^{\sigma-1}[(h-\upsilon)^{\frac{1}{\sigma}} + \epsilon^{\frac{1}{\sigma}}]^{1-\sigma}}{(h-\upsilon)^{\frac{1}{\sigma}}(\sigma-1)} + \frac{[(\Im-\upsilon)^{\frac{1}{\varepsilon}} + \epsilon^{\frac{1}{\varepsilon}}]^{1-\varepsilon} - \epsilon^{\frac{1-\varepsilon}{\varepsilon}}}{(\Im-\upsilon)^{\frac{1}{\varepsilon}}(1-\varepsilon)}.$$

引理3[13] 假设$k_1, k_2, \cdots, k_l \geqslant 0, s_1 > 1$,同时$0 < s_2 < 1$,那么:

$$\sum_{n=1}^{L} k_n^{s_1} \geqslant L^{1-s_1} \cdot \left(\sum_{n=1}^{L} k_n\right)^{s_1}, \sum_{n=1}^{L} k_n^{s_2} \geqslant \left(\sum_{n=1}^{L} k_n\right)^{s_2}.$$

引理4[16] 假设C-正则函数$V(l): \mathcal{R}^{2d} \mapsto \mathcal{R}$,$l(t)$是一个绝对连续函数.对于$[0, +\infty)$每一个紧区间$L$,都有$V(l): \mathcal{R}^{2d} \mapsto \mathcal{R}$是可微的,并且对于$\forall t \in [0, +\infty)$,有:

$$\frac{\mathrm{d}V(l(t))}{\mathrm{d}t} = \mho^{\mathrm{T}}(t)\dot{l}(t), \forall \mho(t) \in \partial V\left(l(t)\right).$$

2 驱动系统(1)和响应系统(3)的固定时间投影同步

本节给出了驱动系统(1)和响应系统(3)之间固定时间投影同步的结果.

定理1 若假设1和假设2成立,$\upsilon < \min\{\Im, h\}$,$\rho_k > \Delta_r + 2LM_r \max\varsigma_{kr} + 2LM_r \max o_{kr} m_r(t)$,那么驱动系统(1)和响应系统(3)在杂合控制器$u_k(t)$的作用下达到固定时间投影同步,计算公式如下:

$$\upsilon = \min_{1 \leqslant k \leqslant L}\{1 - \delta_k - \zeta_k, \varpi_k(t) + \xi_k + \Lambda_k\}, \quad (13)$$

$$\Im = \min_{1 \leqslant k \leqslant L}\{\theta_k, \nu_k\}, \quad (14)$$

$$h = (2L)^{1-\tau} \cdot \min_{1 \leqslant k \leqslant L}\{\gamma_k, \mu_k\}, \quad (15)$$

$$\epsilon = \sum_{k=1}^{L}(\rho_k - \Delta_r - 2LM_r \max\varsigma_{kr} - 2LM_r \max o_{kr} m_r). \quad (16)$$

证明 定义一个C-正则函数:

$$V(t) = \sum_{k=1}^{L}(|\dot{E}_k(t)| + |E_k(t)|),$$

然后对该函数进行求导:

$$\begin{aligned}
\frac{\mathrm{d}V(t)}{\mathrm{d}t} &= \sum_{k=1}^{L}\left\{\dot{E}_k(t) \cdot \phi_k(t) + \ddot{E}_k(t) \cdot \mathrm{R}_k(t)\right\} = \\
&\sum_{k=1}^{L}\{\dot{E}_k(t) \cdot \phi_k(t) + (-\delta_k \frac{\mathrm{d}E_k(t)}{\mathrm{d}t^2} - \varpi_k\left(l_k(t)\right) l_k(t) + \varrho\varpi_k\left(j_k(t)\right) j_k(t) + \\
&\sum_{r=1}^{L}\left[\iota_{kr}\left(l_k\right) F_r\left(l_r(t)\right) - \varrho\iota_{kr}\left(j_k\right) F_r\left(j_r(t)\right)\right] + \\
&\sum_{r=1}^{L}\left[\varsigma_{kr}\left(l_k(t)\right) F_r\left(l_r(t - \tau_r(t))\right) - \varrho\varsigma_{kr}\left(j_k(t)\right) F_r\left(j_r(t - \tau_r(t))\right)\right] + \\
&\sum_{r=1}^{L}\left[\int_{t-m_r(t)}^{t} o_{kr}(t) F_r\left(l_r(t)\right)\mathrm{d}t - \varrho\int_{t-m_r(t)}^{t} o_{kr}(t) F_r\left(j_r(t)\right)\mathrm{d}t\right] + \\
&\left(1 - \varrho\right) I_k(t) u_k(t) \cdot \mathrm{R}_k(t)\} = \\
&\sum_{k=1}^{L}\{\dot{E}_k(t) \cdot \phi_k(t) + \mathrm{R}_k(t)(-\delta_k \frac{\mathrm{d}E_k(t)}{\mathrm{d}t^2} - \varpi_k(t) E_k(t) + \\
&\sum_{r=1}^{L}\left[\iota_{kr}(t) F_r\left(l_r(t)\right) - \iota_{kr}(t) F_r\left(\varrho(j_r(t))\right)\right] + \\
&\sum_{r=1}^{L}\varsigma_{kr}(t)\left[F_r\left(l_r(t - \tau_r(t))\right) - \varrho F_r\left(j_r(t - \tau_r(t))\right)\right] + \\
&\sum_{r=1}^{L} o_{kr}(t)\left[\int_{t-m_r(t)}^{t} F_r\left(l_r(t)\right)\mathrm{d}t - \varrho\int_{t-m_r(t)}^{t} F_r\left(j_r(t)\right)\mathrm{d}t\right] + \\
&\left(1 - \varrho\right) I_k(t) + \tilde{\omega}_k(t)\}.
\end{aligned} \quad (17)$$

在杂合控制器(7)的作用下得到:

$$\begin{aligned}
K\left[\omega_k(t)\right] &= -\xi_k E_k(t) - \zeta_k \dot{E}_k(t) - co\left[\mathrm{sign}\left(\dot{E}_k(t)\right)\right]\left[\rho_k + \theta_k\left|E_k(t)\right|^{\varepsilon} + \nu_k\left|\dot{E}_k(t)\right|^{\varepsilon} + \gamma_k\left|E_k(t)\right|^{\tau} + \mu_k\left|\dot{E}_k(t)\right|^{\tau}\right], \\
\tilde{\omega}_k(t) &= -\xi_k E_k(t) - \zeta_k \dot{E}_k(t) - \mathrm{R}_k(t)\left[\rho_k + \theta_k\left|E_k(t)\right|^{\varepsilon} + \nu_k\left|\dot{E}_k(t)\right|^{\varepsilon} + \gamma_k\left|E_k(t)\right|^{\tau} + \mu_k\left|\dot{E}_k(t)\right|^{\tau}\right].
\end{aligned} \quad (18)$$

根据公式(17)和(18),有:

$$\begin{aligned}
\frac{\mathrm{d}V(t)}{\mathrm{d}t} &= \sum_{k=1}^{L}\{\dot{E}_k(t) \cdot \phi_k(t) + \\
&\mathrm{R}_k(t) \cdot [-(\delta_k + \zeta_k)\dot{E}_k(t) - (\varpi_k + \xi_k) E_k(t) + \\
&\sum_{r=1}^{L}\left[\iota_{kr}(t) F_r\left(l_r(t)\right) - \iota_{kr}(t) F_r\left(\varrho(j_r(t))\right)\right] + \\
&\sum_{r=1}^{L}\varsigma_{kr}(t)\left[F_r\left(l_r(t - \tau_r(t))\right) - \varrho F_r\left(j_r(t - \tau_r(t))\right)\right] + \\
&\sum_{r=1}^{L} o_{kr}(t)\left[\int_{t-m_r(t)}^{t} F_r\left(l_r(t)\right)\mathrm{d}t - \varrho\int_{t-m_r(t)}^{t} F_r\left(j_r(t)\right)\mathrm{d}t\right] + \\
&\left(1 - \varrho\right) I_k - \rho_k - \left|E_k(t)\right|^{\varepsilon}\theta_k - \nu_k\left|\dot{E}_k(t)\right|^{\varepsilon} - \\
&-\gamma_k\left|E_k(t)\right|^{\tau} - \mu_k\left|\dot{E}_k(t)\right|^{\tau}]\} \leqslant
\end{aligned}$$

$$\sum_{k=1}^{L}\{\left|\dot{E}_k(t)\right|\left(1-\delta_k-\zeta_k\right)+\left|E_k(t)\right|\left(\varpi_k(t)+\xi_k+\Lambda_k\right)+ 2LM_r\max\varsigma_{kr}+ 2L\max o_{kr}m_rM_r+\Delta_r-\rho_k-\theta_k\left|E_k(t)\right|^{\varepsilon}-\nu_k\left|\dot{E}_k(t)\right|^{\varepsilon}- \gamma_k\left|E_k(t)\right|^{\tau}-\mu_k\left|\dot{E}_k(t)\right|^{\tau}\}\leqslant$$

$$\upsilon\sum_{k=1}^{L}\left(\left|\dot{E}_k(t)\right|+\left|E_k(t)\right|\right)-\sum_{k=1}^{L}\left(\left|E_k(t)\right|^{\varepsilon}\theta_k+ \nu_k\left|\dot{E}_k(t)\right|^{\varepsilon}+\gamma_k\left|E_k(t)\right|^{\tau}+\mu_k\left|\dot{E}_k(t)\right|^{\tau}\right)-\epsilon\leqslant$$

$$\upsilon\sum_{k=1}^{L}\left(\left|\dot{E}_k(t)\right|+\left|E_k(t)\right|\right)-\theta_k\sum_{k=1}^{L}\left(\left|E_k(t)\right|\right)^{\varepsilon}- \nu_k\sum_{k=1}^{L}\left(\dot{E}_k(t)|\right)^{\varepsilon}-1^{1-\tau}\cdot\gamma_k\left(\sum_{k=1}^{L}\left|E_k(t)\right|\right)^{\tau}-1^{1-\tau}\cdot \mu_k\left(\sum_{k=1}^{L}\left|\dot{E}_k(t)\right|\right)^{\tau}\leqslant\upsilon V(t)-\Im V(t)^{\varepsilon}-hV(t)^{\sigma}-\epsilon.\quad(19)$$

此时,根据引理1可以得到驱动系统(1)和响应系统(3)在控制器(7)的作用下达到固定时间投影同步,另外,镇定时间是$T_{\max}$.

推论1　(1)当投影参数$\varrho\neq0,1,-1$时,驱动系统(1)和响应系统(3)达到固定时间投影同步;

(2)当投影参数$\varrho=1$时,驱动系统(1)和响应系统(3)达到固定时间完全同步;

(3)当投影参数$\varrho=-1$时,驱动系统(1)和响应系统(3)达到固定时间反同步;

(4)当投影参数$\varrho=0$时,驱动系统(1)达到镇定.

注2　实际上,把$\omega_k(t)$替换成$\omega_k^*(t)$,也可以研究预设时间投影同步:

$$\omega_k^*(t)=-\xi_kE_k(t)-\zeta_k\dot{E}_k(t)- \frac{T_{\max}}{T_p}\left\{\left[\left|E_k(t)\right|^{\varepsilon}\theta_k+\nu_k\left|\dot{E}_k(t)\right|^{\varepsilon}+\gamma_k\left|E_k(t)\right|^{\tau}+ \mu_k\left|\dot{E}_k(t)\right|^{\tau}+\epsilon_k+\frac{T_p}{T_{\max}}\mho_k\right]\cdot\mathrm{sign}\left(\dot{E}_k(t)\right)\right\}+ \upsilon\left(\frac{T_{\max}}{T_p}-1\right)\left[\dot{E}_k(t)+\mathrm{sign}\left(\dot{E}_k(t)\right)\cdot\left|E_k(t)\right|\right].$$

3　仿真模拟

例1　如下是二维惯性神经网络:

$$\begin{cases}\dfrac{\mathrm{d}^2j_1(t)}{\mathrm{d}t}=-0.8\dfrac{\mathrm{d}j_1(t)}{\mathrm{d}t}-\varpi_1\left(j_1(t)\right)j_1(t)+\sum_{k=1}^{2}\iota_{1r}\left(j_1(t)\right)F_r\left(j_r(t)\right)+\sum_{k=1}^{2}\varsigma_{1r}\left(j_1(t)\right)F_r\left(j_r\left(t-\tau_r(t)\right)\right)+\\ \qquad\sum_{k=1}^{2}o_{1r}\left(j_1(t)\right)\int_{t-m_r(t)}^{t}F_r\left(j_r(t)\right)\mathrm{d}t,\\ \dfrac{\mathrm{d}^2j_2(t)}{\mathrm{d}t}=-1.4\dfrac{\mathrm{d}j_2(t)}{\mathrm{d}t}-\varpi_2\left(j_2(t)\right)j_2(t)+\sum_{k=1}^{2}\iota_{2r}\left(j_2(t)\right)F_r\left(j_r(t)\right)+\sum_{k=1}^{2}\varsigma_{2r}\left(j_2(t)\right)F_r\left(j_r\left(t-\tau_r(t)\right)\right)+\\ \qquad\sum_{k=1}^{2}o_{2r}\left(j_2(t)\right)\int_{t-m_r(t)}^{t}F_r\left(j_r(t)\right)\mathrm{d}t.\end{cases}\quad(20)$$

系统(20)的不连续激活函数如下:

$$\begin{cases}F_r\left(j_r(t)\right)=\tanh\left(j_r(t)\right)+0.03,j_r(t)\leqslant0,\\ F_r\left(j_r(t)\right)=\tanh\left(j_r(t)\right)-0.04,j_r(t)>0,r=1,2,\end{cases}\quad(21)$$

这里,$\tau_r(t)=1,m_r(t)=\dfrac{\mathrm{e}^t}{1+\mathrm{e}^t}$.

相位图的初始位置为$\phi_1(s)=-0.85,\phi_2(s)=0.7,\varphi_1(s)=0.9,\varphi_2(s)=0.6,\forall s\in[-1,0)$,根据驱动系统(20),响应系统如下:

$$\begin{cases}\dfrac{\mathrm{d}^2l_1(t)}{\mathrm{d}t}=-0.8\dfrac{\mathrm{d}l_1(t)}{\mathrm{d}t}-\varpi_1\left(l_1(t)\right)l_1(t)+\sum_{k=1}^{2}\iota_{1r}\left(l_1(t)\right)F_r\left(l_r(t)\right)+\sum_{k=1}^{2}\varsigma_{1r}\left(l_1(t)\right)F_r\left(l_r\left(t-\tau_r(t)\right)\right)+\\ \qquad\sum_{k=1}^{2}o_{1r}\left(l_1(t)\right)\int_{t-m_r(t)}^{t}F_r\left(l_r(t)\right)\mathrm{d}t+u_1(t),\\ \dfrac{\mathrm{d}^2l_2(t)}{\mathrm{d}t}=-1.4\dfrac{\mathrm{d}l_2(t)}{\mathrm{d}t}-\varpi_2\left(l_2(t)\right)l_2(t)+\sum_{k=1}^{2}\iota_{2r}\left(l_2(t)\right)F_r\left(l_r(t)\right)+\sum_{k=1}^{2}\varsigma_{2r}\left(l_2(t)\right)F_r\left(l_r\left(t-\tau_r(t)\right)\right)+\\ \qquad\sum_{k=1}^{2}o_{2r}\left(l_2(t)\right)\int_{t-m_r(t)}^{t}F_r\left(l_r(t)\right)\mathrm{d}t+u_2(t),\end{cases}\quad(22)$$

其中初始位置为:$\chi_1(s)=-0.1,\chi_2(s)=-0.5,\psi_1(s)=0.9,\psi_2(s)=0.7,\forall s\in[-1,0)$.

根据不连续激活函数(21)可得$M_r=1.04,\Lambda_k=1,\Delta_k=0.07$,假设$\varepsilon=0.5,\tau=1.5,\xi_1=180,\xi_2=190,\zeta_1=\zeta_2=50,o_1=o_2=\nu_1=\nu_2=195,\varpi_1=\varpi_2=\mu_1=\mu_2=390,\rho_1=10.986,\rho_2=10.57$.计算后得$\upsilon=193.95,\Im=h=194.35,\epsilon=0.2$,满足定理1的所有条件.因此,惯性神经网络(20)和(22)在控制器(7)的作用下达到固定时间投影同步,镇定时间为$T_{\max}=8.628$.表2给出了当k=1,2时驱动系统的参数.图1~6为模型(20)与(22)在此参数下的仿真模拟结果.

表2 当k=1,2时的驱动系统的参数

Tab. 2 Parameter values of the driven system when k=1,2

参数	$\lvert j_1(t)\rvert \leqslant 1$	$\lvert j_1(t)\rvert > 1$	参数	$\lvert j_2(t)\rvert \leqslant 1$	$\lvert j_2(t)\rvert > 1$
$\varpi_1(j_1(t))$	0.8	0.85	$\varpi_2(j_2(t))$	0.9	0.95
$\iota_{11}(j_1(t))$	1.8	2	$\iota_{21}(j_2(t))$	1.9	2.1
$\iota_{12}(j_1(t))$	-0.1	-0.08	$\iota_{22}(j_2(t))$	-0.11	-0.09
$\varsigma_{11}(j_1(t))$	-1.3	-1.4	$\varsigma_{21}(j_2(t))$	-1.5	-1.6
$\varsigma_{12}(j_1(t))$	-0.08	-0.1	$\varsigma_{22}(j_2(t))$	-0.09	-0.11
$o_{11}(j_1(t))$	1.6	1.4	$o_{21}(j_2(t))$	1.5	1.45
$o_{12}(j_1(t))$	0.4	0.3	$o_{22}(j_2(t))$	0.3	0.2

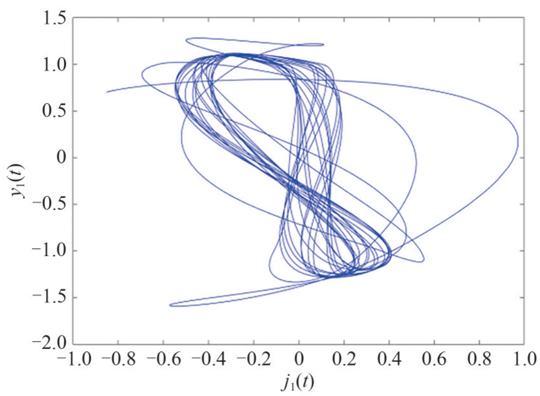

图1 二阶惯性神经网络(20)的二维相图

Fig. 1 Two-dimensional phase diagram of second-order INNs (20)

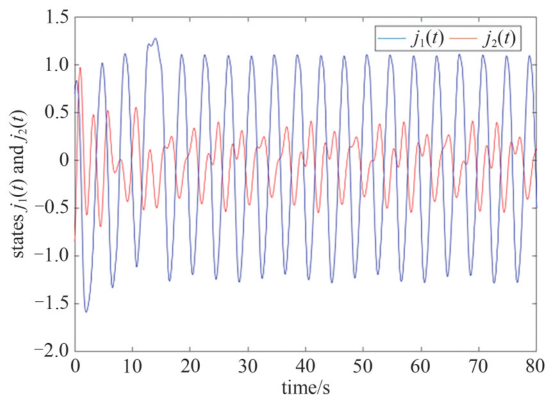

图2 二阶惯性神经网络(20)的状态$j_1(t)$和$j_2(t)$的轨迹图

Fig. 2 Tracks of states $j_1(t)$ and $j_2(t)$ for second-order INNs (20)

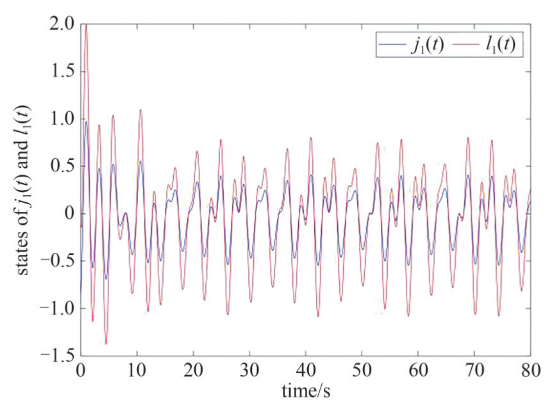

图3 当ϱ = 2时状态$j_1(t)$和$l_1(t)$的固定时间投影同步的轨迹图

Fig. 3 FXPS tracks of states $j_1(t)$ and $l_1(t)$ when ϱ = 2

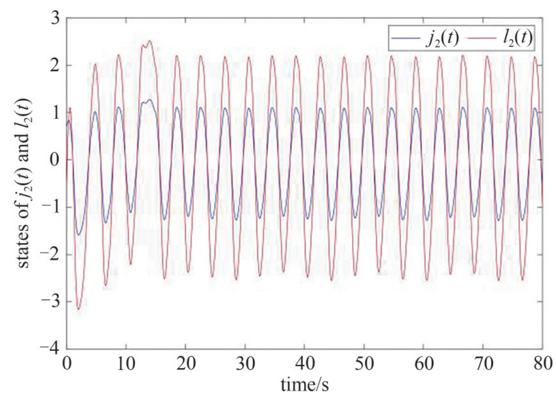

图4 当ϱ = 2时状态$j_2(t)$和$l_2(t)$的固定时间投影同步的轨迹图

Fig. 4 FXPS tracks of states $j_2(t)$ and $l_2(t)$ when ϱ = 2

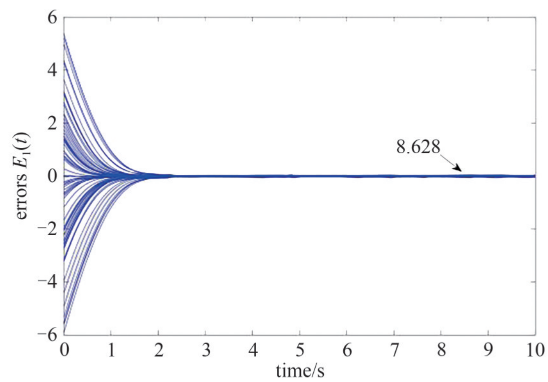

图5 当ϱ = 2时误差系统$E_1(t)$在控制器(7)作用下的状态

Fig. 5 States of error system $E_1(t)$ under controller (7) when ϱ = 2

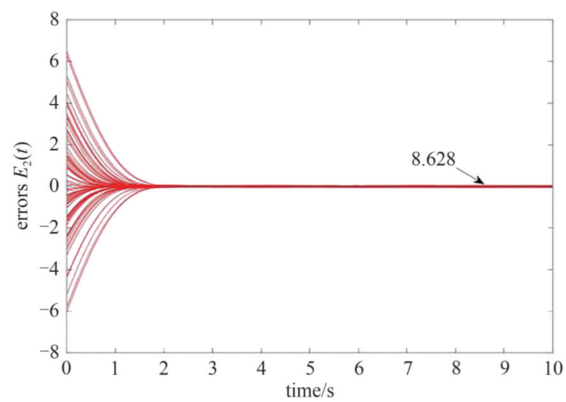

图6 当ϱ = 2时误差系统$E_2(t)$在控制器(7)作用下的状态

Fig. 6 States of error system$E_2(t)$ under controller (7) when ϱ = 2

4 结语

本文利用杂合控制器、微分包含理论、固定时间控制理论、不等式放缩技巧和非降阶法，选取恰当的Lyapunov函数、控制器增益研究了惯性神经网络的固定时间投影同步问题，获得相关判据，并给出模拟仿真例子有效证明结论的有效性．本文所得结论丰富并简化了文献[8]的成果和控制器．

参 考 文 献

[1] ABIODUN O I, KIRU M U, JANTAN A, et al. Comprehensive review of artificial neural network applications to pattern recognition[J]. IEEE Access, 2019, 7: 158820-158846.

[2] SPECHT. Probabilistic neural networks for classification, mapping, or associative memory [C]//IEEE 1988 International Conference on Neural Networks. San Diego: IEEE, 1988: 525-532.

[3] KRIEGESKORTE N. Deep neural networks: A new framework for modeling biological vision and brain information processing[J]. Annual Review of Vision Science, 2015, 1: 417-446.

[4] BIGDELI N, FARID Y, AFSHAR K. A novel image encryption/decryption scheme based on chaotic neural networks[J]. Engineering Applications of Artificial Intelligence, 2012, 25(4): 753-765.

[5] HAN J, CHEN G, WANG L, et al. Direct approach on fixed-time stabilization and projective synchronization of inertial neural networks with mixed delays[J]. Neurocomputing, 2023, 535: 97-106.

[6] ZENG H B, HE Y, WU M, et al. Passivity analysis for neural networks with a time-varying delay[J]. Neurocomputing, 2011, 74(5): 730-734.

[7] SHARMA K, CEREZO M, CINCIO L, et al. Trainability of dissipative perceptron-based quantum neural networks[J]. Physical Review Letters, 2022, 128(18): 180505.

[8] ZHANG G, CAO J, KASHKYNBAYEV A. Further results on fixed/preassigned-time projective lag synchronization control of hybrid inertial neural networks with time delays[J]. Journal of the Franklin Institute, 2023, 360(13): 9950-9973.

[9] LU W, CHEN T. Synchronization of coupled connected neural networks with delays[J]. IEEE Transactions on Circuits and Systems I: Regular Papers, 2004, 51(12): 2491-2503.

[10] WEI W, YU J, WANG L, et al. Fixed/Preassigned-time synchronization of quaternion-valued neural networks via pure power-law control[J]. Neural Networks, 2022, 146: 341-349.

[11] WU A, WEN S, ZENG Z. Synchronization control of a class of memristor-based recurrent neural networks[J]. Information Sciences, 2012, 183(1): 106-116.

[12] ZHANG G, ZENG Z, NING D. Novel results on synchronization for a class of switched inertial neural networks with distributed delays[J]. Information Sciences, 2020, 511: 114-126.

[13] ZHANG G, CAO J. New results on fixed/predefined-time synchronization of delayed fuzzy inertial discontinuous neural networks: Non-reduced order approach[J]. Applied Mathematics and Computation, 2023, 440: 127671.

[14] ZHANG G, ZENG Z. Stabilization of second-order memristive neural networks with mixed time delays *via* nonreduced order[J]. IEEE Transactions on Neural Networks and Learning Systems, 2020, 31(2): 700-706.

[15] KONG F, ZHU Q, HUANG T. New fixed-time stability lemmas and applications to the discontinuous fuzzy inertial neural networks[J]. IEEE Transactions on Fuzzy Systems, 2021, 29(12): 3711-3722.

[16] CLARKE F H, LEDYAEV Y S, STERN R J, et al. Nonsmooth analysis and control theory[M]. New York: Springer New York, 2008.

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