第 46 卷第 6 期
Vol. 46 No. 6 2025

青岛理工大学学报
Journal of Qingdao University of Technology

全状态约束下 CCSTR 随机系统的有限时间控制

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摘　要：针对具有全状态约束的串级连续搅拌反应釜随机系统提出有限时间控制方法。利用模糊逻辑系统处理非线性函数与随机扰动；采用命令滤波技术避免“计算爆炸”的问题，引入误差补偿机制降低误差对系统的影响；利用有限时间技术实现对信号的快速和精准跟踪；通过结合 Itô 随机微分方程和障碍 Lyapunov 函数对系统进行稳定性分析，以确保系统的所有状态不违反约束边界；MATLAB 仿真表明，该方法能够在全状态约束下克服随机扰动的影响，实现对串级连续搅拌反应釜随机系统的有限时间快速跟踪控制。

关键词：串级连续搅拌反应釜；随机系统；全状态约束；命令滤波控制；有限时间控制
中图分类号：TP273.2　　**文献标志码**：A　　**文章编号**：1673-4602(2025)06-0123-11

Finite-time control of a stochastic system in CCSTR under full-state constraint

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Abstract: A finite-time control method is proposed for the stochastic system in cascade continuous stirred tank reactors (CCSTR) with full-state constraint. The fuzzy logic systems are used to handle nonlinear functions and random disturbances. The technique of command filtered control is used to avoid the problem of “computational explosion”, and the error compensation mechanism is introduced to reduce the impact of the error on the system. The finite-time technology is utilized to achieve fast and accurate tracking of signals. The stability of the system is analyzed by combining the Itô stochastic differential equation and the barrier Lyapunov function to ensure that all states of the system do not violate the constraint boundary. MATLAB simulations show that the proposed method can overcome the influence of random disturbance under the full-state constraint and realize the finite-time fast-tracking control of the stochastic system of CCSTR.

Key words: cascade continuous stirred tank reactors (CCSTR); stochastic system; full-state constraint; command filtered control; finite-time control

收稿日期：2023-12-25
基金项目：山东省自然科学基金（ZR2025MS1101；ZR2021MF076）；山东省重点研发项目（2018GHY115025）；中国博士后面上项目（2018M642611）
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串级连续搅拌反应釜(Cascade Continuous Stirred Tank Reactors，CCSTR)在化学反应装置中占有重要地位，并广泛应用于生物、制药和化学过程，但CCSTR的强耦合性、不确定性和复杂非线性使其精准控制变得困难[1]。为了获得良好的控制性能，提出了神经网络、命令滤波、有限时间等控制策略[2-6]。然而，现有的控制策略在应对CCSTR系统的随机性特征方面还存在不足。随机扰动[7]在实际工业控制系统中是普遍存在的，而CCSTR系统的反应过程往往涉及多相反应相互转换，对釜内的随机扰动十分敏感[8-9]。因此，开发一种能够更有效地应对CCSTR系统随机特性的控制算法是当前的研究重点。

目前尚未有针对CCSTR随机系统控制策略的报道，但对于单个连续搅拌反应釜(Continuous Stirring Tank Reactor，CSTR)随机系统已经开发了一些控制策略，如针对单输入单输出不确定随机非线性系统，提出了随机有限时间控制方法，并成功应用于CSTR[10]；针对一类P型切换随机非线性系统的有限时间稳定问题，设计了状态反馈控制法[11]。因此，本文基于有限时间命令滤波控制为CCSTR系统设计了应对系统自身随机特性的控制器。考虑到随机扰动易引起系统状态变量大幅度变化，影响控制器的控制精度，在控制器设计时有必要将系统整个响应过程的所有状态变量约束在合理的范围内[12-13]。

为了解决考虑全状态约束的CCSTR随机系统的精准控制问题，本文设计了一种全状态约束下CCSTR随机系统的有限时间控制算法。利用模糊逻辑系统处理非线性函数与随机扰动；采用命令滤波技术解决“计算爆炸”的问题；引入误差补偿机制降低误差对系统的影响；利用有限时间技术实现随机系统对信号的快速和精准跟踪；通过结合Itô随机微分方程和障碍Lyapunov函数(Barrier Lyapunov Function，BLF)对系统进行稳定性分析，以确保系统的所有状态不违反约束边界。与现有控制器相比，本文所设计的控制器优点如下：①针对CCSTR系统的随机扰动特性，提出了全状态约束下CCSTR随机系统有限时间命令滤波控制算法；②构造BLF以保证全状态约束，确保了系统所有状态不违反约束边界。

1　系统描述和预备知识

1.1　系统描述

在2个相互串联的反应器Ⅰ与Ⅱ内进行不可逆的放热反应，为使反应器内部的温度维持在一个恒定的参考值，在2个反应器外设置冷却夹套。假设2个反应器的体积关系为 $V_1 = V_2 = V$，2个冷却夹套的体积关系为 $V_{j1} = V_{j2} = V_j$，反应物流速关系为 $Q_0 = Q_2 = Q$，$Q_1 = Q_R + Q$。CCSTR系统结构如图1所示。

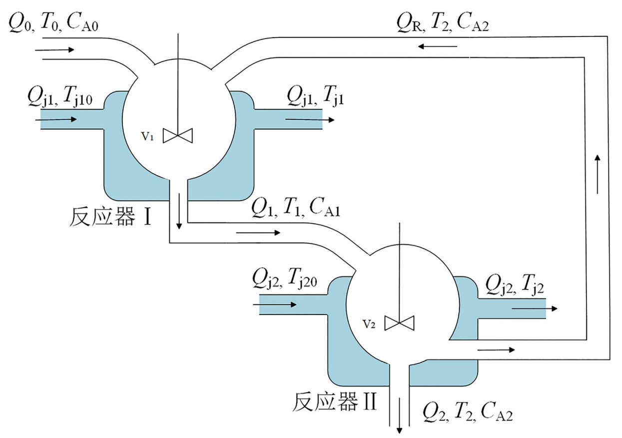

图1　CCSTR系统结构

T_0—反应釜Ⅰ中反应物的进口温度；T_1、T_2—反应釜Ⅰ、Ⅱ的温度；T_{j10}、T_{j1}—反应釜Ⅰ冷却剂的进出口温度；T_{j20}、T_{j2}—反应釜Ⅱ冷却剂的进出口温度；C_{A0}—反应釜Ⅰ的进料浓度；C_{A1}、C_{A2}—反应釜Ⅰ、Ⅱ中的物料浓度

随机系统是一个典型的不确定系统，为了建立随机稳定性，考虑随机非线性系统：

$$d\boldsymbol{x} = f(\boldsymbol{x},\boldsymbol{u})dt + g^{T}(\boldsymbol{x},\boldsymbol{u})d\boldsymbol{\omega} \tag{1}$$

式中：$\boldsymbol{x} \in \mathbf{R}^N$ 为系统状态向量；$\boldsymbol{u} \in \mathbf{R}^m$ 为系统的输入信号；$f(\cdot)$ 和 $g(\cdot)$ 分别为已知的光滑的连续函数，在 x 上连续且满足 $f(0)=g(0)=0$；$\boldsymbol{\omega}$ 为独立的 r 维Wiener过程。

为了简化CCSTR模型以及设计和计算过程，CCSTR的数学模型表达式为

$$\begin{cases} \dot{x}_{11} = b_{11}x_{12}dt + \boldsymbol{\psi}_{11}^{T}d\omega, & \dot{x}_{12} = b_{12}u_1 dt + \boldsymbol{\psi}_{12}^{T}d\omega, & y_1 = x_{11} \\ \dot{x}_{21} = (b_{21}x_{22} + \phi_{21} + \boldsymbol{\Phi}x_{31})dt + \boldsymbol{\psi}_{21}^{T}d\omega, & \dot{x}_{22} = (b_{22}u_2 + \phi_{22})dt + \boldsymbol{\psi}_{22}^{T}d\omega, & y_2 = x_{21} \\ \dot{x}_{31} = (b_{31}x_{32} + \phi_{31} + \boldsymbol{\Psi}w)dt + \boldsymbol{\psi}_{31}^{T}d\omega, & \dot{x}_{32} = (b_{32}u_3 + \phi_{32})dt + \boldsymbol{\psi}_{32}^{T}d\omega, & y_3 = x_{31} \end{cases} \tag{2}$$

式中：$\boldsymbol{\psi}_{11} = \sin x_{11}$，$\boldsymbol{\psi}_{12} = \cos x_{12}$，$\boldsymbol{\psi}_{21} = \sin x_{21}$，$\boldsymbol{\psi}_{22} = \cos x_{22}$，$\boldsymbol{\psi}_{31} = \sin x_{31}$，$\boldsymbol{\psi}_{32} = \cos x_{32}$ 为扰动函数；其余参数与文献[5]一致。

注1　本文的控制目标是设计随机系统式(2)的控制器，使输出信号 y_i 在有限时间内快速收敛至期

望的参考信号 x_{id}，并且闭环系统中 x_{ij} 的状态都被约束在 $\Omega_x := \{|x_{ij}| < k_{cij}(i=1,2,3; j=1,2)\}$ 内，其中 $k_{cij} > 0$。

1.2　预备知识

定义 1　对于 $V=V(x)\in C^2$，C^2 表示复数集，由 Itô 微分法则，定义微分运算 L 如下

$$LV=\frac{\partial V}{\partial x}f+\frac{1}{2}\mathrm{Tr}\left\{\boldsymbol{h}^{\mathrm{T}}\frac{\partial^2 V}{\partial x^2}\boldsymbol{h}\right\} \tag{3}$$

其中，$\mathrm{Tr}\left\{\boldsymbol{h}^{\mathrm{T}}\frac{\partial^2 V}{\partial x^2}\boldsymbol{h}\right\}$ 表示矩阵 $\boldsymbol{h}^{\mathrm{T}}\frac{\partial^2 V}{\partial x^2}\boldsymbol{h}$ 的迹。

引理 1[14]　非线性连续函数 $f(\boldsymbol{X})$ 被模糊逻辑系统逼近，可以写为

$$f(\boldsymbol{X})=\boldsymbol{W}^{\mathrm{T}}\boldsymbol{S}(\boldsymbol{X})+\varepsilon \tag{4}$$

$$\boldsymbol{S}(\boldsymbol{X})=[P_1(\boldsymbol{X}),P_2(\boldsymbol{X}),\cdots,P_N(\boldsymbol{X})]^{\mathrm{T}}/\sum_{i=1}^{N}P_i(\boldsymbol{X})$$

式中：$\boldsymbol{S}(\boldsymbol{X})$ 为模糊基函数向量；$P_i(\boldsymbol{X})$ 为基函数；$N>1$ 为模糊规则的数量；$\boldsymbol{W}\in\mathbf{R}^N$ 为最优权向量；ε 为逼近误差，对于任意 $\bar{\varepsilon}>0$，$|\varepsilon|\leqslant\bar{\varepsilon}$。

基函数 $P_i(\boldsymbol{X})$ 用高斯函数表示为

$$P_i(\boldsymbol{X})=\exp\left[\frac{-(\boldsymbol{X}-\boldsymbol{\mu}_i)^{\mathrm{T}}(\boldsymbol{X}-\boldsymbol{\mu}_i)}{2\eta_i^2}\right] \tag{5}$$

式中：$\boldsymbol{\mu}_i=[\mu_{i1},\mu_{i2},\cdots,\mu_{iN}]$ 为中心向量；η_i 为高斯函数宽度。

2　随机系统有限时间命令滤波控制器设计

运用 BLF 确保全状态约束，结合 Itô 随机微分方程为 CCSTR 随机系统设计了有限时间命令滤波控制器，设计步骤如下：

步骤 1　定义跟踪误差为 $z_{11}=x_{11}-x_{1d}$，$z_{12}=x_{12}-x_{1,c}$；命令滤波器的补偿跟踪误差为 $v_{11}=z_{11}-\zeta_{11}$，$v_{12}=z_{12}-\zeta_{12}$。其中，$x_{1d}=0$ 为参考信号，$x_{1,c}$ 为命令滤波器的输出。

选取 BLF 为 $V_{11}=\frac{1}{4b_{11}}\log\frac{k_{b11}^4}{k_{b11}^4-v_{11}^4}$，由 Itô 公式可得

$$LV_{11}=\frac{v_{11}^3}{k_{b11}^4-v_{11}^4}(v_{12}+\zeta_{12}+x_{1,c}-\dot{\zeta}_{11}+\alpha_{11}-\alpha_{11})+\frac{v_{11}^2}{2(k_{b11}^4-v_{11}^4)^2}(3k_{b11}^4+v_{11}^4)\boldsymbol{\psi}_{11}^{\mathrm{T}}\boldsymbol{\psi}_{11} \tag{6}$$

由杨氏不等式可得

$$\frac{v_{11}^3}{k_{b11}^4-v_{11}^4}v_{12}\leqslant\frac{3}{4}v_{11}^4(k_{b11}^4-v_{11}^4)^{-\frac{4}{3}}+\frac{1}{4}v_{12}^4 \tag{7}$$

$$\frac{v_{11}^2}{2(k_{b11}^4-v_{11}^4)^2}(3k_{b11}^4+v_{11}^4)\boldsymbol{\psi}_{11}^{\mathrm{T}}\boldsymbol{\psi}_{11}\leqslant\frac{v_{11}^3}{k_{b11}^4-v_{11}^4}\left[\frac{v_{11}(3k_{b11}^4-v_{11}^4)^2\|\boldsymbol{\psi}_{11}\|^4}{4J_{11}^2(k_{b11}^4-v_{11}^4)^3}\right]+\frac{1}{4}J_{11}^2 \tag{8}$$

式中：$k_{b11}>0$ 为设计参数；J_{11} 为正常数。

定义未知函数

$$f_{11}(\boldsymbol{X}_{11})=\frac{v_{11}(3k_{b11}^4-v_{11}^4)^2\|\boldsymbol{\psi}_{11}\|^4}{4J_{11}^2(k_{b11}^4-v_{11}^4)^3} \tag{9}$$

基于模糊逻辑系统的普遍逼近特性，$f_{11}(\boldsymbol{X}_{11})=\boldsymbol{W}_{11}^{\mathrm{T}}\boldsymbol{S}_{11}(\boldsymbol{X}_{11})+\varepsilon_{11}$，由杨氏不等式可得

$$\frac{v_{11}^3}{k_{b11}^4-v_{11}^4}f_{11}\leqslant\frac{v_{11}^6}{2l_{11}^2(k_{b11}^4-v_{11}^4)^2}\|\boldsymbol{W}_{11}\|^2\boldsymbol{S}_{11}^{\mathrm{T}}\boldsymbol{S}_{11}+\frac{1}{2}l_{11}^2+\frac{3}{4}v_{11}^4(k_{b11}^4-v_{11}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{11}^4 \tag{10}$$

$$\frac{v_{11}^3h_{11}\mathrm{sign}(\zeta_{11})}{k_{b11}^4-v_{11}^4}\leqslant\frac{3}{4}v_{11}^4(k_{b11}^4-v_{11}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{11}^4 \tag{11}$$

选取虚拟控制函数、误差补偿函数为

$$\alpha_{11}=-k_{11}z_{11}-\frac{3}{2}v_{11}(k_{b11}^4-v_{11}^4)^{-\frac{1}{3}}-\frac{s_{11}v_{11}^{4\gamma-3}}{(k_{b11}^4-v_{11}^4)^{\gamma-1}}-\frac{v_{11}^3}{2l_{11}^2(k_{b11}^4-v_{11}^4)}\hat{\theta}\boldsymbol{S}_{11}^{\mathrm{T}}\boldsymbol{S}_{11} \tag{12}$$

$$\dot{\zeta}_{11}=-k_{11}\zeta_{11}+\zeta_{12}+x_{1,c}-\alpha_{11}-h_{11}\operatorname{sign}(\zeta_{11}) \tag{13}$$

式中：$l_{11}>0$，$h_{11}>0$，$k_{11}>0$，$s_{11}>0$，$0<\gamma<1$ 为设计参数；$\hat{\theta}$ 为自适应率 θ 的估计值。

将式(7)—(13)代入式(6)可得

$$LV_{11}\leqslant -k_{11}\frac{v_{11}^4}{k_{b11}^4-v_{11}^4}-s_{11}\left(\frac{v_{11}^4}{k_{b11}^4-v_{11}^4}\right)^{\gamma}+\frac{1}{2}l_{11}^2+\frac{1}{4}\bar{\varepsilon}_{11}^4+\frac{v_{11}^6}{2l_{11}^2(k_{b11}^4-v_{11}^4)^2}(\|\boldsymbol{W}_{11}\|^2-\hat{\theta})\boldsymbol{S}_{11}^{\mathrm{T}}\boldsymbol{S}_{11}+\frac{1}{4}h_{11}^4+\frac{1}{4}J_{11}^2+\frac{1}{4}v_{12}^4 \tag{14}$$

步骤2 选取 BLF 为 $V_{12}=V_{11}+\frac{1}{4b_{12}}\log\frac{k_{b12}^4}{k_{b12}^4-v_{12}^4}$，由 Itô 公式可得

$$LV_{12}=LV_{11}+\frac{v_{12}^3}{k_{b12}^4-v_{12}^4}(u_1-\dot{x}_{1,c}-\dot{\zeta}_{12})+\frac{v_{12}^2}{2(k_{b12}^4-v_{12}^4)^2}(3k_{b12}^4+v_{12}^4)\boldsymbol{\psi}_{12}^{\mathrm{T}}\boldsymbol{\psi}_{12} \tag{15}$$

由杨氏不等式可得

$$\frac{v_{12}^2}{2(k_{b12}^4-v_{12}^4)^2}(3k_{b12}^4+v_{12}^4)\boldsymbol{\psi}_{12}^{\mathrm{T}}\boldsymbol{\psi}_{12}\leqslant\frac{v_{12}^3}{k_{b12}^4-v_{12}^4}\left(\frac{v_{12}(3k_{b12}^4-v_{12}^4)^2\|\boldsymbol{\psi}_{12}\|^4}{4J_{12}^2(k_{b12}^4-v_{12}^4)^3}\right)+\frac{1}{4}J_{12}^2 \tag{16}$$

式中：$k_{b12}>0$ 为设计参数；J_{12} 为正常数。

定义未知函数

$$f_{12}(\boldsymbol{X}_{12})=\dot{x}_{1,c}+\frac{v_{12}(3k_{b12}^4-v_{12}^4)^2\|\boldsymbol{\psi}_{12}\|^4}{4J_{12}^2(k_{b12}^4-v_{12}^4)^3} \tag{17}$$

基于模糊逻辑系统的普遍逼近特性，$f_{12}(\boldsymbol{X}_{12})=\boldsymbol{W}_{12}^{\mathrm{T}}\boldsymbol{S}_{12}(\boldsymbol{X}_{12})+\varepsilon_{12}$，由杨氏不等式可得

$$\frac{v_{12}^3}{k_{b12}^4-v_{12}^4}f_{12}\leqslant\frac{v_{12}^6}{2l_{12}^2(k_{b12}^4-v_{12}^4)^2}\|\boldsymbol{W}_{12}\|^2\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{12}+\frac{1}{2}l_{12}^2+\frac{3}{4}v_{12}^4(k_{b12}^4-v_{12}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{12}^4 \tag{18}$$

$$\frac{v_{12}^3h_{12}\operatorname{sign}(\zeta_{12})}{k_{b12}^4-v_{12}^4}\leqslant\frac{3}{4}v_{12}^4(k_{b12}^4-v_{12}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{12}^4 \tag{19}$$

选取实际控制函数、误差补偿函数为

$$u_1=-k_{12}z_{12}-\frac{1}{4}z_{12}-\frac{3}{2}v_{12}(k_{b12}^4-v_{12}^4)^{-\frac{1}{3}}-\frac{s_{12}v_{12}^{4\gamma-3}}{(k_{b12}^4-v_{12}^4)^{\gamma-1}}-\frac{v_{12}^3}{2l_{12}^2(k_{b12}^4-v_{12}^4)}\hat{\theta}\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{12} \tag{20}$$

$$\dot{\zeta}_{12}=-k_{12}\zeta_{12}-\frac{1}{4}\zeta_{12}-h_{12}\operatorname{sign}(\zeta_{12}) \tag{21}$$

式中：$l_{12}>0$，$h_{12}>0$，$k_{12}>0$，$s_{12}>0$ 为设计参数。

将式(16)—(21)代入式(15)可得

$$LV_{12}\leqslant-\sum_{i=1}^{1}\sum_{j=1}^{2}k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{1}\sum_{j=1}^{2}s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}+\sum_{i=1}^{1}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\sum_{i=1}^{1}\sum_{j=1}^{2}\frac{1}{2l_{ij}^2}\frac{v_{ij}^6}{(k_{bij}^4-v_{ij}^4)^2}(\|\boldsymbol{W}_{ij}\|^2-\hat{\theta})\boldsymbol{S}_{ij}^{\mathrm{T}}\boldsymbol{S}_{ij} \tag{22}$$

步骤3 定义跟踪误差为 $z_{21}=x_{21}-x_{2d}$，$z_{22}=x_{22}-x_{2,c}$；命令滤波器的补偿跟踪误差为 $v_{21}=z_{21}-\zeta_{21}$，$v_{22}=z_{22}-\zeta_{22}$。其中，$x_{2d}=0$ 为参考信号；$x_{2,c}$ 为命令滤波器的输出。

选取 BLF 为 $V_{21}=V_{12}+\frac{1}{4b_{21}}\log\frac{k_{b21}^4}{k_{b21}^4-v_{21}^4}$，由 Itô 公式可得

$$LV_{21}=LV_{12}+\frac{v_{21}^3}{k_{b21}^4-v_{21}^4}\left(v_{22}+\zeta_{22}+x_{2,c}+\frac{\phi_{21}}{b_{21}}+\frac{\Phi x_{31}}{b_{21}}-\frac{\dot{x}_{2d}}{b_{21}}-\frac{\dot{\zeta}_{21}}{b_{21}}+\alpha_{21}-\alpha_{21}\right)+\frac{v_{21}^2}{2(k_{b21}^4-v_{21}^4)^2}(3k_{b21}^4+v_{21}^4)\boldsymbol{\psi}_{21}^{\mathrm{T}}\boldsymbol{\psi}_{21} \tag{23}$$

由杨氏不等式可得

$$\frac{v_{21}^3}{k_{b21}^4-v_{21}^4}v_{22}\leqslant\frac{3}{4}v_{21}^4(k_{b21}^4-v_{21}^4)^{-\frac{4}{3}}+\frac{1}{4}v_{22}^4 \tag{24}$$

$$\frac{v_{21}^2}{2(k_{b21}^4-v_{21}^4)^2}(3k_{b21}^4+v_{21}^4)\boldsymbol{\psi}_{21}^{\mathrm{T}}\boldsymbol{\psi}_{21}\leqslant\frac{v_{21}^3}{k_{b21}^4-v_{21}^4}\left[\frac{v_{21}(3k_{b21}^4-v_{21}^4)^2\|\boldsymbol{\psi}_{21}\|^4}{4J_{21}^2(k_{b21}^4-v_{21}^4)^3}\right]+\frac{1}{4}J_{21}^2 \tag{25}$$

式中：$k_{b21}>0$ 为设计参数；J_{21} 为正常数。

定义未知函数

$$f_{21}(\boldsymbol{X}_{21})=\frac{\phi_{21}}{b_{21}}+\frac{\Phi x_{31}}{b_{21}}+\frac{v_{21}(3k_{b21}^4-v_{21}^4)^2\|\boldsymbol{\psi}_{21}\|^4}{4J_{21}^2(k_{b21}^4-v_{21}^4)^3} \tag{26}$$

基于模糊逻辑系统的普遍逼近特性，$f_{21}(\boldsymbol{X}_{21})=\boldsymbol{W}_{21}^{\mathrm{T}}\boldsymbol{S}_{21}(\boldsymbol{X}_{21})+\varepsilon_{21}$，由杨氏不等式可得

$$\frac{v_{21}^3}{k_{b21}^4-v_{21}^4}f_{21}\leqslant\frac{v_{21}^6}{2l_{21}^2(k_{b21}^4-v_{21}^4)^2}\|\boldsymbol{W}_{21}\|^2\boldsymbol{S}_{21}^{\mathrm{T}}\boldsymbol{S}_{21}+\frac{1}{2}l_{21}^2+\frac{3}{4}v_{21}^4(k_{b21}^4-v_{21}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{21}^4 \tag{27}$$

$$\frac{v_{21}^3h_{21}\mathrm{sign}(\zeta_{21})}{k_{b21}^4-v_{21}^4}\leqslant\frac{3}{4}v_{21}^4(k_{b21}^4-v_{21}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{21}^4 \tag{28}$$

选取虚拟控制函数、误差补偿函数为

$$\alpha_{21}=-k_{21}z_{21}-\frac{3}{2}v_{21}(k_{b21}^4-v_{21}^4)^{-\frac{1}{3}}-\frac{s_{21}v_{21}^{4\gamma-3}}{(k_{b21}^4-v_{21}^4)^{\gamma-1}}-\frac{v_{21}^3}{2l_{21}^2(k_{b21}^4-v_{21}^4)}\hat{\theta}\boldsymbol{S}_{21}^{\mathrm{T}}\boldsymbol{S}_{21} \tag{29}$$

$$\dot{\zeta}_{21}=b_{21}(-k_{21}\zeta_{21}+\zeta_{22}+x_{2,c}-\alpha_{21}-h_{21}\mathrm{sign}(\zeta_{21})) \tag{30}$$

式中：$l_{21}>0$，$h_{21}>0$，$k_{21}>0$，$s_{21}>0$ 为设计参数。

将式(24)—(30)代入式(23)可得

$$LV_{21}\leqslant LV_{12}-k_{21}\frac{v_{21}^4}{k_{b21}^4-v_{21}^4}-s_{21}\left(\frac{v_{21}^4}{k_{b21}^4-v_{21}^4}\right)^{\gamma}+\frac{1}{2}l_{21}^2+\frac{1}{4}\bar{\varepsilon}_{21}^4+$$
$$\frac{v_{21}^6}{2l_{21}^2(k_{b21}^4-v_{21}^4)^2}(\|\boldsymbol{W}_{21}\|^2-\hat{\theta})\boldsymbol{S}_{21}^{\mathrm{T}}\boldsymbol{S}_{21}+\frac{1}{4}h_{21}^4+\frac{1}{4}J_{21}^2+\frac{1}{4}v_{22}^4 \tag{31}$$

步骤 4　选取 BLF 为 $V_{22}=V_{21}+\frac{1}{4b_{22}}\log\frac{k_{b22}^4}{k_{b22}^4-v_{22}^4}$，由 Itô 公式可得

$$LV_{22}=LV_{21}+\frac{v_{22}^3}{k_{b22}^4-v_{22}^4}\left(u_2+\frac{\phi_{22}}{b_{22}}-\frac{\dot{x}_{2,c}}{b_{22}}-\frac{\dot{\zeta}_{22}}{b_{22}}\right)+\frac{v_{22}^2}{2(k_{b22}^4-v_{22}^4)^2}(3k_{b22}^4+v_{22}^4)\boldsymbol{\psi}_{22}^{\mathrm{T}}\boldsymbol{\psi}_{22} \tag{32}$$

由杨氏不等式可得

$$\frac{v_{22}^2}{2(k_{b22}^4-v_{22}^4)^2}(3k_{b22}^4+v_{22}^4)\boldsymbol{\psi}_{22}^{\mathrm{T}}\boldsymbol{\psi}_{22}\leqslant\frac{v_{22}^3}{k_{b22}^4-v_{22}^4}\left[\frac{v_{22}(3k_{b22}^4-v_{22}^4)^2\|\boldsymbol{\psi}_{22}\|^4}{4J_{22}^2(k_{b22}^4-v_{22}^4)^3}\right]+\frac{1}{4}J_{22}^2 \tag{33}$$

式中：$k_{b22}>0$ 为设计参数；J_{22} 为正常数。

定义未知函数

$$f_{22}(\boldsymbol{X}_{22})=\frac{\phi_{22}}{b_{22}}-\frac{\dot{x}_{2,c}}{b_{22}}+\frac{v_{22}(3k_{b22}^4-v_{22}^4)^2\|\boldsymbol{\psi}_{22}\|^4}{4J_{22}^2(k_{b22}^4-v_{22}^4)^3} \tag{34}$$

基于模糊逻辑系统的普遍逼近特性，$f_{22}(\boldsymbol{X}_{22})=\boldsymbol{W}_{22}^{\mathrm{T}}\boldsymbol{S}_{22}(\boldsymbol{X}_{22})+\varepsilon_{22}$，由杨氏不等式可得

$$\frac{v_{22}^3}{k_{b22}^4-v_{22}^4}f_{22}\leqslant\frac{v_{22}^6}{2l_{22}^2(k_{b22}^4-v_{22}^4)^2}\|\boldsymbol{W}_{22}\|^2\boldsymbol{S}_{22}^{\mathrm{T}}\boldsymbol{S}_{22}+\frac{1}{2}l_{22}^2+\frac{3}{4}v_{22}^4(k_{b22}^4-v_{22}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{22}^4 \tag{35}$$

$$\frac{v_{22}^3h_{22}\mathrm{sign}(\zeta_{22})}{k_{b22}^4-v_{22}^4}\leqslant\frac{3}{4}v_{22}^4(k_{b22}^4-v_{22}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{22}^4 \tag{36}$$

选取实际控制函数、误差补偿函数为

$$u_2=-k_{22}z_{22}-\frac{1}{4}z_{22}-\frac{3}{2}v_{22}(k_{b22}^4-v_{22}^4)^{-\frac{1}{3}}-\frac{s_{22}v_{22}^{4\gamma-3}}{(k_{b22}^4-v_{22}^4)^{\gamma-1}}-\frac{v_{22}^3}{2l_{22}^2(k_{b22}^4-v_{22}^4)}\hat{\theta}\boldsymbol{S}_{22}^{\mathrm{T}}\boldsymbol{S}_{22} \tag{37}$$

$$\dot{\zeta}_{22}=b_{22}(-k_{22}\zeta_{22}-\frac{1}{4}\zeta_{22}-h_{22}\mathrm{sign}(\zeta_{22})) \tag{38}$$

式中：$l_{22}>0, h_{22}>0, k_{22}>0, s_{22}>0$ 为设计参数。

将式(33)—(38)代入式(32)可得

$$LV_{22}\leqslant -\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{2}\sum_{j=1}^{2}s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}+\sum_{i=1}^{2}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{1}{2l_{ij}^2}\frac{v_{ij}^6}{(k_{bij}^4-v_{ij}^4)^2}(\|\boldsymbol{W}_{ij}\|^2-\hat{\theta})\boldsymbol{S}_{ij}^{\mathrm{T}}\boldsymbol{S}_{ij} \tag{39}$$

步骤5 定义跟踪误差为 $z_{31}=x_{31}-x_{3d}, z_{32}=x_{32}-x_{3,c}$；命令滤波器的补偿跟踪误差 $v_{31}=z_{31}-\zeta_{31}$，$v_{32}=z_{32}-\zeta_{32}$。其中，$x_{3d}=0$ 为参考信号，$x_{3,c}$ 为命令滤波器的输出。

选取 BLF 为 $V_{31}=V_{22}+\frac{1}{4b_{31}}\log\frac{k_{b31}^4}{k_{b31}^4-v_{31}^4}$，由 Itô 公式可得

$$LV_{31}=LV_{22}+\frac{v_{31}^3}{k_{b31}^4-v_{31}^4}\left(v_{32}+\zeta_{32}+x_{3,c}+\frac{\phi_{31}+\Psi w}{b_{31}}-\frac{\dot{x}_{3d}}{b_{31}}-\frac{\dot{\zeta}_{31}}{b_{31}}+\alpha_{31}-\alpha_{31}\right)+\frac{v_{31}^2}{2(k_{b31}^4-v_{31}^4)^2}(3k_{b31}^4+v_{31}^4)\boldsymbol{\psi}_{31}^{\mathrm{T}}\boldsymbol{\psi}_{31} \tag{40}$$

由杨氏不等式可得

$$\frac{v_{31}^3}{k_{b31}^4-v_{31}^4}v_{32}\leqslant\frac{3}{4}v_{31}^4(k_{b31}^4-v_{31}^4)^{-\frac{4}{3}}+\frac{1}{4}v_{32}^4 \tag{41}$$

$$\frac{v_{31}^2}{2(k_{b31}^4-v_{31}^4)^2}(3k_{b31}^4+v_{31}^4)\boldsymbol{\psi}_{31}^{\mathrm{T}}\boldsymbol{\psi}_{31}\leqslant\frac{v_{31}^3}{k_{b31}^4-v_{31}^4}\left[\frac{v_{31}(3k_{b31}^4-v_{31}^4)^2\|\boldsymbol{\psi}_{31}\|^4}{4J_{31}^2(k_{b31}^4-v_{31}^4)^3}\right]+\frac{1}{4}J_{31}^2 \tag{42}$$

式中：$k_{b31}>0$ 为设计参数；J_{31} 为正常数。

定义未知函数

$$f_{31}(\boldsymbol{X}_{31})=\frac{\phi_{31}+\Psi w}{b_{31}}+\frac{v_{31}(3k_{b31}^4-v_{31}^4)^2\|\boldsymbol{\psi}_{31}\|^4}{4J_{31}^2(k_{b31}^4-v_{31}^4)^3} \tag{43}$$

基于模糊逻辑系统的普遍逼近特性，$f_{31}(\boldsymbol{X}_{31})=\boldsymbol{W}_{31}^{\mathrm{T}}\boldsymbol{S}_{31}(\boldsymbol{X}_{31})+\varepsilon_{31}$，由杨氏不等式可得

$$\frac{v_{31}^3}{k_{b31}^4-v_{31}^4}f_{31}\leqslant\frac{v_{31}^6}{2l_{31}^2(k_{b31}^4-v_{31}^4)^2}\|\boldsymbol{W}_{31}\|^2\boldsymbol{S}_{31}^{\mathrm{T}}\boldsymbol{S}_{31}+\frac{1}{2}l_{31}^2+\frac{3}{4}v_{31}^4(k_{b31}^4-v_{31}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{31}^4 \tag{44}$$

$$\frac{v_{31}^3h_{31}\operatorname{sign}(\zeta_{31})}{k_{b31}^4-v_{31}^4}\leqslant\frac{3}{4}v_{31}^4(k_{b31}^4-v_{31}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{31}^4 \tag{45}$$

选取虚拟控制函数、误差补偿函数为

$$\alpha_{31}=-k_{31}z_{31}-\frac{3}{2}v_{31}(k_{b31}^4-v_{31}^4)^{-\frac{1}{3}}-\frac{s_{31}v_{31}^{4\gamma-3}}{(k_{b31}^4-v_{31}^4)^{\gamma-1}}-\frac{v_{31}^3}{2l_{31}^2(k_{b31}^4-v_{31}^4)}\hat{\theta}\boldsymbol{S}_{31}^{\mathrm{T}}\boldsymbol{S}_{31} \tag{46}$$

$$\dot{\zeta}_{31}=b_{31}(-k_{31}\zeta_{31}+\zeta_{32}+x_{3,c}-\alpha_{31}-h_{31}\operatorname{sign}(\zeta_{31})) \tag{47}$$

式中：$l_{31}>0, h_{31}>0, k_{31}>0, s_{31}>0$ 为设计参数。

将式(41)—(47)代入式(40)可得

$$LV_{31}\leqslant LV_{22}-k_{31}\frac{v_{31}^4}{k_{b31}^4-v_{31}^4}-s_{31}\left(\frac{v_{31}^4}{k_{b31}^4-v_{31}^4}\right)^{\gamma}+\frac{1}{2}l_{31}^2+\frac{1}{4}\bar{\varepsilon}_{31}^4+\frac{v_{31}^6}{2l_{31}^2(k_{b31}^4-v_{31}^4)^2}(\|\boldsymbol{W}_{31}\|^2-\hat{\theta})\boldsymbol{S}_{31}^{\mathrm{T}}\boldsymbol{S}_{31}+\frac{1}{4}h_{31}^4+\frac{1}{4}J_{31}^2+\frac{1}{4}v_{32}^4 \tag{48}$$

步骤6 选取 BLF 为 $V_{32}=V_{31}+\frac{1}{4b_{32}}\log\frac{k_{b32}^4}{k_{b32}^4-v_{32}^4}$，由 Itô 公式可得

$$LV_{32}=LV_{31}+\frac{v_{32}^3}{k_{b32}^4-v_{32}^4}\left(u_3+\frac{\phi_{32}}{b_{32}}-\frac{\dot{x}_{3,c}}{b_{32}}-\frac{\dot{\zeta}_{32}}{b_{32}}\right)+\frac{v_{32}^2}{2(k_{b32}^4-v_{32}^4)^2}(3k_{b32}^4+v_{32}^4)\boldsymbol{\psi}_{32}^{\mathrm{T}}\boldsymbol{\psi}_{32} \tag{49}$$

由杨氏不等式可得

$$\frac{v_{32}^2}{2(k_{b32}^4-v_{32}^4)^2}(3k_{b32}^4+v_{32}^4)\boldsymbol{\psi}_{32}^{\mathrm{T}}\boldsymbol{\psi}_{32}\leqslant\frac{v_{32}^3}{k_{b32}^4-v_{32}^4}\left[\frac{v_{32}(3k_{b32}^4-v_{32}^4)^2\|\boldsymbol{\psi}_{32}\|^4}{4J_{32}^2(k_{b32}^4-v_{32}^4)^3}\right]+\frac{1}{4}J_{32}^2 \tag{50}$$

式中:$k_{b32}>0$ 为设计参数;J_{32} 为正常数。

定义未知函数

$$f_{32}(\boldsymbol{X}_{32})=\frac{\phi_{32}}{b_{32}}+\frac{v_{32}(3k_{b32}^4-v_{32}^4)^2\|\boldsymbol{\psi}_{32}\|^4}{4J_{32}^2(k_{b32}^4-v_{32}^4)^3} \tag{51}$$

基于模糊逻辑系统的普遍逼近特性,$f_{32}(\boldsymbol{X}_{32})=\boldsymbol{W}_{32}^{\mathrm{T}}\boldsymbol{S}_{32}(\boldsymbol{X}_{32})+\varepsilon_{32}$,由杨氏不等式可得

$$\frac{v_{32}^3}{k_{b32}^4-v_{32}^4}f_{32}\leqslant\frac{v_{32}^6}{2l_{32}^2(k_{b32}^4-v_{32}^4)^2}\|\boldsymbol{W}_{32}\|^2\boldsymbol{S}_{32}^{\mathrm{T}}\boldsymbol{S}_{32}+\frac{1}{2}l_{32}^2+\frac{3}{4}v_{32}^4(k_{b32}^4-v_{32}^4)^{-\frac{4}{3}}+\frac{1}{4}\bar{\varepsilon}_{32}^4 \tag{52}$$

$$\frac{v_{32}^3h_{32}\operatorname{sign}(\zeta_{32})}{k_{b32}^4-v_{32}^4}\leqslant\frac{3}{4}v_{32}^4(k_{b32}^4-v_{32}^4)^{-\frac{4}{3}}+\frac{1}{4}h_{32}^4 \tag{53}$$

选取实际控制函数、误差补偿函数为

$$u_3=-k_{32}z_{32}-\frac{1}{4}z_{32}-\frac{3}{2}v_{32}(k_{b32}^4-v_{32}^4)^{-\frac{1}{3}}-\frac{s_{32}v_{32}^{4\gamma-3}}{(k_{b32}^4-v_{32}^4)^{\gamma-1}}-\frac{v_{32}^3}{2l_{32}^2(k_{b32}^4-v_{32}^4)}\hat{\theta}\boldsymbol{S}_{32}^{\mathrm{T}}\boldsymbol{S}_{32} \tag{54}$$

$$\dot{\zeta}_{32}=b_{32}(-k_{32}\zeta_{32}-\frac{1}{4}\zeta_{32}-h_{32}\operatorname{sign}(\zeta_{32})) \tag{55}$$

式中:$l_{32}>0,h_{32}>0,k_{32}>0,s_{32}>0$ 为设计参数。

将式(50)—(55)代入式(49)可得

$$\begin{aligned}LV_{32}\leqslant&-\sum_{i=1}^{3}\sum_{j=1}^{2}k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{3}\sum_{j=1}^{2}s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}+\\&\sum_{i=1}^{3}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\sum_{i=1}^{3}\sum_{j=1}^{2}\frac{1}{2l_{ij}^2}\frac{v_{ij}^6}{(k_{bij}^4-v_{ij}^4)^2}(\|\boldsymbol{W}_{ij}\|^2-\hat{\theta})\boldsymbol{S}_{ij}^{\mathrm{T}}\boldsymbol{S}_{ij}\end{aligned} \tag{56}$$

式中:θ 为未知常数,定义 $\theta=\max\{\|\boldsymbol{W}_{11}\|,\|\boldsymbol{W}_{21}\|,\|\boldsymbol{W}_{31}\|\}$。

3　稳定性分析

定义 Lyapunov 函数 $V_n=V_{32}+\frac{1}{2r_1}\tilde{\theta}^2$,$\tilde{\theta}=\theta-\hat{\theta}$。由 Itô 公式可得

$$\begin{aligned}LV_n\leqslant&-\sum_{i=1}^{3}\sum_{j=1}^{2}k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{3}\sum_{j=1}^{2}s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}+\\&\sum_{i=1}^{3}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\frac{1}{r_1}\tilde{\theta}\left[\dot{\hat{\theta}}-\sum_{i=1}^{3}\sum_{j=1}^{2}\frac{r_1}{2l_{ij}^2}\frac{v_{ij}^6}{(k_{bij}^4-v_{ij}^4)^2}\boldsymbol{S}_{ij}^{\mathrm{T}}\boldsymbol{S}_{ij}\right]\end{aligned} \tag{57}$$

式中:$k_{bij}>0,r_1>0$ 为设计参数;$\dot{\hat{\theta}}$ 为自适应律,假设 $\dot{\hat{\theta}}=\dot{\tilde{\theta}}$,构造自适应律 $\dot{\hat{\theta}}$ 为

$$\dot{\hat{\theta}}=-m_1\hat{\theta}+\sum_{i=1}^{3}\sum_{j=1}^{2}\frac{r_1}{2l_{ij}^2}\frac{v_{ij}^6}{(k_{bij}^4-v_{ij}^4)^2}\boldsymbol{S}_{ij}^{\mathrm{T}}\boldsymbol{S}_{ij} \tag{58}$$

式中:$m_1>0$ 为设计参数。

将式(58)代入式(57)可得

$$\begin{aligned}LV_n\leqslant&-\sum_{i=1}^{3}\sum_{j=1}^{2}k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{3}\sum_{j=1}^{2}s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}+\\&\sum_{i=1}^{3}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)-\frac{m_1}{r_1}\tilde{\theta}\hat{\theta}\end{aligned} \tag{59}$$

由杨氏不等式可得

$$\frac{m_1}{r_1}\tilde{\theta}\hat{\theta}\leqslant-\frac{m_1\tilde{\theta}^2}{4r_1}-\frac{m_1\tilde{\theta}^2}{4r_1}+\frac{m_1\theta^2}{2r_1} \tag{60}$$

$$-\frac{1}{4r_1}\tilde{\theta}^2\leqslant(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}-\left(\frac{1}{4r_1}\tilde{\theta}^2\right)^{\gamma} \tag{61}$$

将式(60)、式(61)代入式(59)可得

$$LV_n \leqslant -\sum_{i=1}^{3}\sum_{j=1}^{2} k_{ij}\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}-\sum_{i=1}^{3}\sum_{j=1}^{2} s_{ij}\left(\frac{v_{ij}^4}{k_{bij}^4-v_{ij}^4}\right)^{\gamma}-m_1\left(\frac{1}{4r_1}\tilde{\theta}^2\right)^{\gamma}+\sum_{i=1}^{3}\sum_{j=1}^{2}\left(\frac{1}{4}h_{ij}^4+\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\frac{m_1}{2r_1}\theta^2-\frac{m_1}{4r_1}\tilde{\theta}^2+m_1(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} \leqslant -aV_n+bV_n^{\gamma}+c \tag{62}$$

其中，$a=\min\{4k_{11},4k_{12},4k_{21},4k_{22},4k_{31},4k_{32},\frac{m_1}{2}\}$；$b=\min\{4^{\gamma}s_{11},4^{\gamma}s_{12},4^{\gamma}s_{21},4^{\gamma}s_{22},4^{\gamma}s_{31},4^{\gamma}s_{32},\frac{m_1}{2^{\gamma}}\}$；$c=\sum_{i=2}^{3}\sum_{j=1}^{2}\left(\frac{1}{2}l_{ij}^2+\frac{1}{4}\bar{\varepsilon}_{ij}^4+\frac{1}{4}J_{ij}^2\right)+\frac{1}{2}l_{12}^2+\frac{1}{4}\bar{\varepsilon}_{12}^4+\frac{m_1}{2r_1}\theta^2+m_1(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}$。

根据文献[15]可以得出 v_{ij}、$\tilde{\theta}$ 将收敛到如下区域

$$(v_{ij},\tilde{\theta})\in\left\{V_n\leqslant\min\left\{\frac{c}{(1-\theta)a}=P_1,\frac{c}{(1-\theta)b}=Q_1\right\}\right\} \tag{63}$$

在有限时间内 $T_1\leqslant\max\{\frac{c}{\theta_0 a(1-\gamma)}\ln\frac{\theta_0 aV_n^{1-\gamma}(0)+b}{b},\frac{1}{a(1-\gamma)}\ln\frac{\theta_0 aV_n^{1-\gamma}(0)+\theta_0 b}{\theta_0 b}\}$。

由于 $z_{ij}=v_{ij}+\zeta_{ij}$，因此有必要证明 ζ_{ij} 在有限时间内是有界的。选取 Lyapunov 函数为 $V_m=\sum_{i=1}^{3}\sum_{j=1}^{2}\frac{1}{2b_{ij}}\zeta_{ij}^2$，由 Itô 公式可得

$$LV_m\leqslant-(k_{11}-1)\zeta_{11}^2-\left(k_{12}-\frac{5}{4}\right)\zeta_{12}^2-\left(k_{21}-\frac{1}{2}\right)\zeta_{21}^2-\left(k_{22}-\frac{3}{4}\right)\zeta_{22}^2-(k_{31}-1)\zeta_{31}^2-\left(k_{32}-\frac{3}{4}\right)\zeta_{32}^2-\sum_{i=1}^{2}\sum_{j=1}^{3}|\zeta_{ij}|h_{ij}+\frac{1}{2}\sum_{i=1}^{3}O_i(\Xi_i^{\rho}) \leqslant -k_0V_m-h_0V_m^{\frac{1}{2}}+\eta_0 \tag{64}$$

其中，$k_0=\min\{2k_{11}-2,2k_{12}-\frac{5}{2},2k_{21}-1,2k_{22}-\frac{3}{2},2k_{31}-2,2k_{32}-\frac{3}{2}\}$，$h_0=\min\{\sqrt{2}h_{11},\sqrt{2}h_{12},\sqrt{2}h_{21},\sqrt{2}h_{22},\sqrt{2}h_{31},\sqrt{2}h_{32}\}$，$\eta_0=\frac{1}{2}\sum_{i=1}^{3}O_i(\Xi_i^{\rho})$。

根据文献[15]可以得出，ζ_{ij} 将收敛到如下区域

$$\zeta_{ij}\in\left\{V_m\leqslant\min\left\{\frac{\eta_0}{(1-\theta)k_0}=P_2,\left[\frac{\eta_0}{(1-\theta)h_0}\right]^{\frac{1}{2}}=Q_2\right\}\right\} \tag{65}$$

在有限时间内 $T_2\leqslant\max\{\frac{2}{\theta k_0}\ln\frac{\theta k_0V_m^{\frac{1}{2}}(0)+h_0}{h_0},\frac{2}{k_0}\ln\frac{k_0V_m^{\frac{1}{2}}(0)+\theta h_0}{\theta h_0}\}$。

由式(63)和式(65)可得

$$\begin{cases}\frac{1}{4}\log\left(\frac{k_{b11}^4}{k_{b11}^4-v_{11}^4}\right)\leqslant\min\{P_1,Q_1\}\\ \frac{1}{2}\zeta_{11}^2\leqslant\min\{P_2,Q_2\}\end{cases} \tag{66}$$

式中：P_1、Q_1、P_2、Q_2 为常数。

因此，

$$\begin{cases}|v_{11}|\leqslant\min\{k_{b11}\sqrt[4]{1-e^{-4P_1}},k_{b11}\sqrt[4]{1-e^{-4Q_1}}\}\\ |\zeta_{11}|\leqslant\min\{\sqrt{2P_2},\sqrt{2Q_2}\}\end{cases} \tag{67}$$

对于 $T\geqslant\max\{T_1,T_2\}$，考虑 $z_{ij}=v_{ij}+\zeta_{ij}$，可得

$$|z_{11}|\leqslant\min\left\{k_{b11}\sqrt[4]{1-e^{-4P_1}},k_{b11}\sqrt[4]{1-e^{-4Q_1}}\right\}+\min\{\sqrt{2P_2},\sqrt{2Q_2}\} \tag{68}$$

因此，在有限时间内，跟踪误差能够收敛到原点附近的小范围内，且可以保证所有信号和全状态约束的有界性。

4　仿真结果及分析

4.1　预备知识

利用 MATLAB 进行模拟仿真，验证 CCSTR 随机系统在全状态约束下的有限时间控制策略的有效性。CSTR 模型的参数取值与文献[6]一致。

选择 CCSTR 系统的状态约束范围为

$$|x_{11}|<k_{c11}=2e^{-x}+0.1,\ |x_{12}|<k_{c12}=16e^{-x}+0.6,\ |x_{21}|<k_{c21}=5e^{-x}+0.25,$$
$$|x_{22}|<k_{c22}=30e^{-x}+1,\ |x_{31}|<k_{c31}=2e^{-x}+0.07,\ |x_{32}|<k_{c32}=25e^{-x}+0.9。$$

选择状态变量的初始值为

$$x_{11}(0)=2, x_{12}(0)=-2; x_{21}(0)=5, x_{22}(0)=-4; x_{31}(0)=2, x_{32}(0)=-4; \hat{\theta}(0)=0.1。$$

选择合适的模糊集为

$$P_1(x)=e^{-\frac{(x+2)^2}{2}}, P_2(x)=e^{-\frac{(x+1)^2}{2}}, P_3(x)=e^{-\frac{(x+0)^2}{2}}, P_4(x)=e^{-\frac{(x-1)^2}{2}}, P_5(x)=e^{-\frac{(x-2)^2}{2}}。$$

全状态约束下 CCSTR 随机系统的有限时间命令滤波控制器的设计参数，如表 1 所示。

表 1　控制器的设计参数值

参数	值	参数	值	参数	值	参数	值
k_{11}	6	l_{31}	1	k_{b21}	5.5	ω_1	7000
k_{12}	200	l_{32}	1	k_{b22}	4.5	ω_2	7000
k_{21}	1	s_{11}	10^{-16}	k_{b31}	2.5	ω_3	7000
k_{22}	30	s_{12}	10^{-12}	k_{b32}	4.5	r_1	0.01
k_{31}	15	s_{21}	10^{-12}	h_{11}	10^{-11}	m_1	10
k_{32}	15	s_{22}	10^{-12}	h_{12}	10^{-11}	γ	0.5
l_{11}	1	s_{31}	10^{-9}	h_{21}	10^{-11}	ζ	0.99
l_{12}	1	s_{32}	10^{-9}	h_{22}	10^{-11}		
l_{21}	10	k_{b11}	2.5	h_{31}	10^{-8}		
l_{22}	10	k_{b12}	2.5	h_{32}	10^{-8}		

4.2　仿真结果分析

为了验证本文方法的可靠性与优越性，在相同的控制器参数条件下，将文献[6]中的有限时间命令滤波控制方法与本文方法进行了对比仿真。系统仿真结果，如图 2—7 所示。

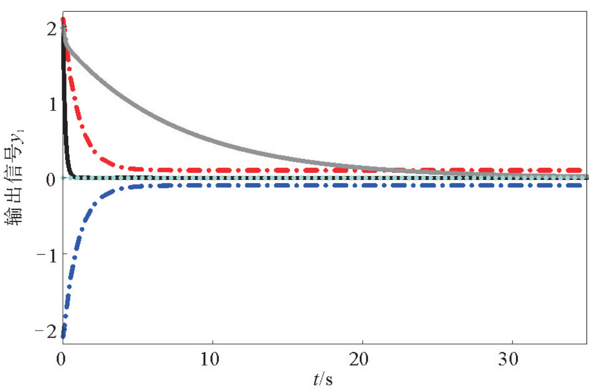

图 2　信号y_1与y_1'对比

k_{c11}；y_1；y_1'；$-k_{c11}$；y_d

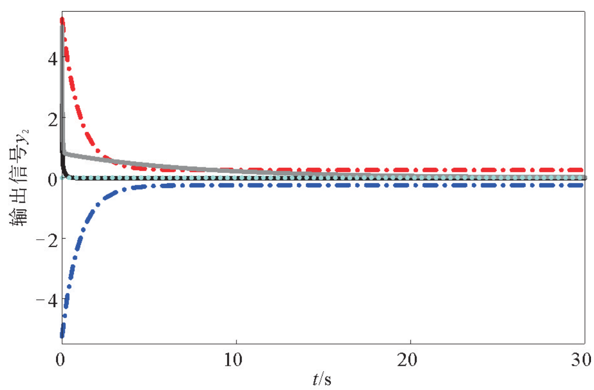

图 3　信号y_1与y_2'对比

k_{c21}；y_2；y_2'；$-k_{c21}$；y_d

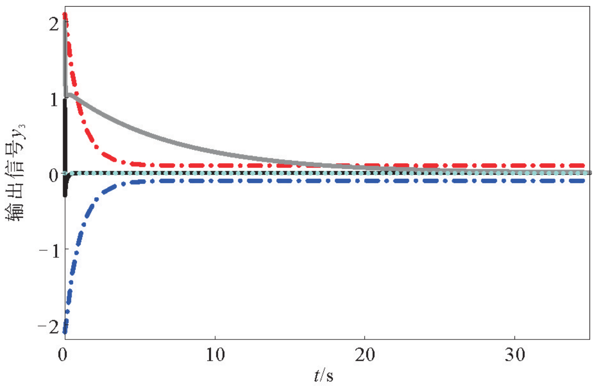

图 4 信号y_1与y_2'对比

k_{c31}; y_3; y_3'; $-k_{c31}$; y_d

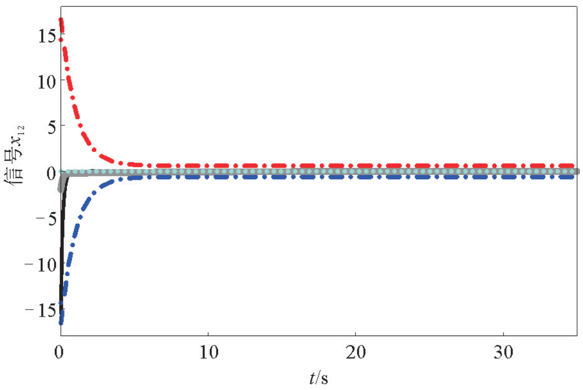

图 5 信号x_{12}与x_{12}'对比

k_{c12}; x_{12}; x_{12}'; $-k_{c12}$; x_{1d}

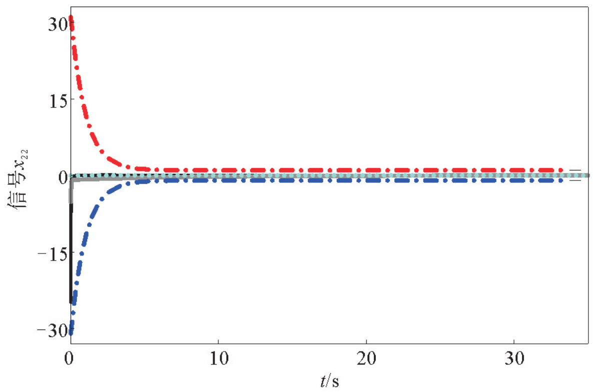

图 6 信号x_{22}与x_{22}'对比

k_{c22}; x_{22}; x_{22}'; $-k_{c22}$; x_{2d}

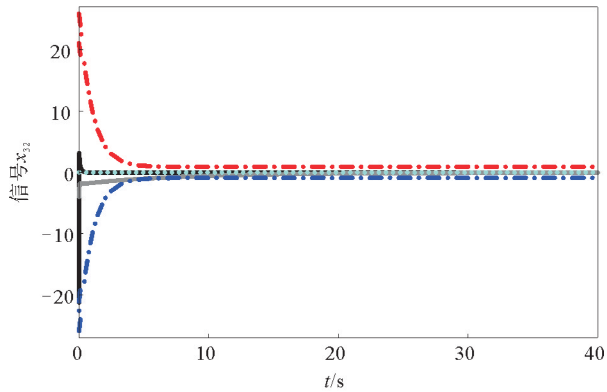

图 7 信号x_{32}与x_{32}'对比

k_{c32}; x_{32}; x_{32}'; $-k_{c32}$; x_{3d}

图 2—7 显示了本文方法与文献[6]方法的跟踪参考信号和状态变量的对比过程。结果表明，针对 CCSTR 系统本文方法在克服随机扰动的同时能够确保系统中所有信号均有界且都被约束在给定范围内，能够更快地使系统状态收敛到参考信号，获得了更为精确和稳定的控制效果。

5 结论

针对 CCSTR 系统的随机特性提出了全状态约束下的有限时间命令滤波控制。该方法不仅有效解决了 CCSTR 系统受随机特性影响的问题，实现了有限时间快速跟踪控制，而且将系统的所有状态约束在给定的范围内，保证了 CCSTR 系统实际生产过程的稳定性和安全性，提供了更为稳定、安全的控制策略。

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(责任编辑　赵金环;英文校审　周立俭)